Falling film over a porous medium

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Falling film over a porous medium: 2D model

\[ L = \text{Macroscopic length scale, } l_\beta, l_\sigma = \text{Microscopic length scale}. \]
Applications of these type of problems:

- Transport phenomena.
- Heat and mass transfer
- Technological processes.
- Coating Technology.
- Filtration process.
- To remove particles from the liquid.
- Fundamental problem in fluid mechanics.
- Transition from order to disorder state (Laminar to turbulence)
Problems specification: Brief description

- Falling film over a slippery substrate.
- Flow is driven by N-S equations: \( \frac{\rho d\mathbf{v}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g} \).
- Slip b.c. \( \mathbf{v} = \alpha \partial_y u \) at the plane.
- Free surface b.c. \( -p \mathbf{l} + (\mathbf{T} \cdot \mathbf{n}) \cdot \mathbf{n} = \sigma \nabla_s \cdot \mathbf{n} \).
Problems specification: Brief description

- Falling film over a porous medium: One-domain approach
- Flow is driven by modified N-S equations
\[
\frac{\rho}{\varepsilon(y)} \left[ \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \left( \frac{\mathbf{v}}{\varepsilon(y)} \right) \right] = -\nabla p + \frac{\mu}{\varepsilon(y)} \nabla^2 \mathbf{v} - \frac{\mu \mathbf{v}}{\kappa(y)} + \rho g.
\]
- No slip b.c. \( \mathbf{v} = 0 \) at the bottom plane.
Problems specification : Brief description

- Falling film over a porous medium: Two-domain approach
- Liquid and porous layers flow driven by N-S and Brinkman eqns.
- Liquid-porous interface b.c.

\[ \boldsymbol{v} = \tilde{\boldsymbol{v}}, \, \partial_y \tilde{u}/\varepsilon - \partial_y u = \beta/\sqrt{\kappa} \tilde{u}, \quad -\tilde{p} + 2\mu/\varepsilon \partial_y \tilde{v} = -p + 2\mu \partial_y v \]
Falling film over a slippery substrate: Basic idea

Darcy law:

\[ \nabla \cdot \mathbf{v} = 0 \]
\[ \nabla p = -\left(\frac{\mu}{\kappa}\right)v + \rho g. \]

Interfacial b.c. (Beavers and Joseph JFM (1967)):

\[ \partial_y u = \frac{\chi}{\sqrt{\kappa}}(u - u_P) \Rightarrow u = \frac{\sqrt{\kappa}}{\chi} \partial_y u \quad (u_P \ll u) \]

Navier-slip b.c.
Falling film over a slippery substrate: Basic idea

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Navier-slip b.c.

\[ \alpha = b\left(\frac{\mu}{\mu_b} - 1\right), \text{ slip length} \]

Blake Colloids and Surfaces (1990)
Falling film over a slippery plane

- **Introduction**

Falling film over a slippery plane

• **Introduction**

• **Motivation**
  • Extend this model for low to moderate values of Reynolds number.
  • Detail study of linear and nonlinear wave dynamics.
  • Influence of slip length on the wave dynamics.
  • Study the influence of streamwise viscous diffusion.
Falling film over a slippery plane

- **Introduction**

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- **Formulation**
  - Orr-Sommerfeld BVP
  - Two-equation model in terms of film thickness $h(x, t)$, flow rate $q(x, t)$.
  - Travelling wave solutions.
  - DNS computation.

- **Conclusions**
Reference scale:

\[ l_\nu = \nu^{2/3}(g \sin \theta)^{-1/3} \], length scale \( t_\nu = \nu^{1/3}(g \sin \theta)^{-2/3} \), time scale.
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Dimensionless numbers:
\[ \Gamma = \frac{\sigma}{\rho(g \sin \theta)^{1/3} \nu^{4/3}}, \] Kapitza number, \[ We = \frac{\Gamma}{h_N^2}, \] Weber number,
\[ Re = \frac{h_N^3}{2(1 + 2\alpha_1)} \] Reynolds number, \[ \alpha_1 = \frac{\alpha}{h_N}, \] slip length,
\[ h_N \] dimensionless Nusselt film thickness, \[ U_N = h_N^2(1/2 + \alpha_1), \] free surface Nusselt velocity.
Marginal stability curve : Temporal stability

When $\Gamma = 2431.0$ and $\theta = 4^\circ$

**Critical Reynolds number**

$$\text{Re}_c = \frac{5}{4} \cot \theta (1 + 3\alpha_1)(1 + 2\alpha_1)/\left[1 + 6\alpha_1 + \left(\frac{25}{2}\right)\alpha_1^2 + \left(\frac{15}{2}\right)\alpha_1^3\right] \Rightarrow \frac{5}{4} \cot \theta \text{ at } \alpha_1 \to 0 \text{ (Benjamin JFM (1957) and Yih Phys. Fluids (1963)).}$$
Two-equation model : BL approximation \( (\partial_x, \partial_t \sim O(\varepsilon)) \)

\[
\begin{align*}
\partial_x u + \partial_y v &= 0, \\
(\partial_t u + u \partial_x u + \nu \partial_y u) &= (2\partial_{xx} u + \partial_x [\partial_x u|_h]) - \cot \theta \partial_x h \\
&\quad + \Gamma \partial_{xxx} h + \partial_{yy} u + 1,
\end{align*}
\]

\[
\begin{align*}
u|_{y=0} &= 0, \\
\partial_y u|_h &= (-\partial_x v + 4\partial_x u \partial_x h)|_h
\end{align*}
\]

\Rightarrow \text{BL-equation } (\text{Chang et al. JFM (1993)}).
Two-equation model : BL approximation ($\partial_x, \partial_t \sim O(\epsilon)$)

$$
\partial_x u + \partial_y v = 0,
$$

$$(\partial_t u + u \partial_x u + v \partial_y u) = (2\partial_{xx} u + \partial_x [\partial_x u |_h]) - \cot \theta \partial_x h + \Gamma \partial_{xxx} h + \partial_{yy} u + 1,$$

$$
u |_{y=0} = 0, \quad \nabla |_{y=0} = \alpha \partial_y u |_{y=0},
$$

$$
\partial_y u |_h = (-\partial_x v + 4\partial_x u \partial_x h) |_h
$$

$\Rightarrow$ BL-equation (Chang et al. JFM (1993)).

Assumption : Streamwise velocity profile

IBL approach (Shkadov Izv. Akad. Mekh. Zhidk. Gaza (1967)) :

$$(i) \ u = u_0 = q/(hF_0)(\bar{y} - \bar{y}^2/2 + \alpha/h), \ F_0 = (1/3 + \alpha/h), \ \bar{y} = y/h \Rightarrow Re_c = 3 \cot \theta g(\alpha_1).$$
Two-equation model: BL approximation \((\partial_x, \partial_t \sim \mathcal{O}(\epsilon))\)

\[
\begin{align*}
\partial_x u + \partial_y v &= 0, \\
(\partial_t u + u \partial_x u + \nu \partial_y u) &= (2\partial_{xx} u + \partial_x [\partial_x u|_h]) - \cot \theta \partial_x h \\
&\quad + \Gamma \partial_{xxx} h + \partial_{yy} u + 1, \\
\left. u \right|_{y=0} &= \alpha \partial_y u|_{y=0}, \\
\left. v \right|_{y=0} &= 0, \\
\left. \partial_y u \right|_{h} &= (-\partial_x v + 4 \partial_x u \partial_x h)|_h
\end{align*}
\]

\Rightarrow BL\text{-equation} \text{ (Chang et al. JFM (1993)).}

Assumption: Streamwise velocity profile


\((i)\) \[ u = u_0 = \frac{q}{(h F_0)}(\bar{y} - \bar{y}^2/2 + \alpha/h), \quad F_0 = (1/3 + \alpha/h), \quad \bar{y} = y/h \]

\Rightarrow Re_c = 3 \cot \theta g(\alpha_1).

WIBL(Galerkin) approach (Ruyer-Quil and Manneville Eur. Phys. J. B (2000)):

\((ii)\) \[ u = u_0 + u_1, \quad \text{where} \int_0^h u_1 dy = 0. \]
Two-equation model:

\[
[\partial_t + u_0 \partial_x + v_0 \partial_y] u_0 + \mathcal{O}(\geq \epsilon^2) = (2 \partial_{xx} u_0 + \partial_x [\partial_x u_0 | h]) - \cot \theta \partial_x h
\]

\[
+ \Gamma \partial_{xxx} h + \partial_{yy} u_0 + \partial_{yy} u_1 + 1.
\]
Two-equation model:

\[
\left[ \partial_t + u_0 \partial_x + v_0 \partial_y \right] u_0 + \mathcal{O}(\geq \epsilon^2) = (2 \partial_{xx} u_0 + \partial_x [\partial_x u_0 | h]) - \cot \theta \partial_x h + \Gamma \partial_{xxx} h + \partial_{yy} u_0 + \partial_{yy} u_1 + 1.
\]

Step I: Multiply weight function \( w(x, t) \) and integrate over the depth \([0 \ h]\)

\[
\int_0^h \partial_{yy} u_1 w dy = \langle w | Lu_1 \rangle = \langle L^\dagger w | u_1 \rangle \quad \text{use adjoint b.c. and } \langle 1 | u_1 \rangle = 0
\]

\( Lw = \text{const.} \Rightarrow w = u_0 \)

where \( L = \partial_{yy} \), \( \langle u | v \rangle = \int_0^h uv dy \)
Two-equation model:

\[
\partial_t + u_0 \partial_x + v_0 \partial_y u_0 + \mathcal{O}(\geq \epsilon^2) = (2\partial_{xx} u_0 + \partial_x [\partial_x u_0 | h]) - \cot \theta \partial_x h \\
+ \Gamma \partial_{xxx} h + \partial_{yy} u_0 + \partial_{yy} u_1 + 1.
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Step I: Multiply weight function \( w(x, t) \) and integrate over the depth \([0, h]\)

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\]

\( Lw = \text{const.} \Rightarrow w = u_0 \)

where \( L = \partial_{yy}, \langle u | v \rangle = \int_0^h uvdy \)

Step II: Integrate mass conservation equation and use kinematic b.c.
Two-equation model

Averaged mass conservation equation

\[ \partial_t h + \partial_x q = 0, \]

Averaged momentum conservation equation

\[ \left[ \partial_t q + F(\alpha/h)q/h \partial_x q - G(\alpha/h)q^2/h^2 \partial_x h \right] = I(\alpha/h)\left[ -(1/F_0(\alpha/h))q/h^2 \\
+ h(1 - \cot \theta \partial_x h + \Gamma \partial_{xxx} h) \right] \\
+ \left[ J(\alpha/h)q/h^2(\partial_x h)^2 - K(\alpha/h)\partial_x q \partial_x h/h - L(\alpha/h)q/h \partial_{xx} h + M(\alpha/h)\partial_{xx} q \right], \]

streamwise viscous diffusion
Problem 1

Linear stability (Whitham wave hierarchy):

Linearized $h = 1 + \hat{h}$, $q = (1/3 + \alpha_1) + \hat{q}$.

Eliminating $\hat{q}$ \Rightarrow

$$
\left( \partial_t + c_k \partial_x \right) \hat{h} + \Omega(Re) \left( \partial_t + c_{d+} \partial_x \right) \left( \partial_t + c_{d-} \partial_x \right) \hat{h} = 0.
$$

KW

DW

Stability condition: $c_{d-} < c_k < c_{d+}$

$$
c_k = c_{d+} \Rightarrow
$$

$$
Re_c = (5/4) \cot \theta (1 + 3\alpha_1)(1 + 2\alpha_1) / [1 + 6\alpha_1 + (25/2)\alpha_1^2 + (15/2)\alpha_1^3]
$$
Two-equation model: Spatial stability

\[ Re = 20 \quad \text{and} \quad Re = 80 \]

When \( \Gamma = 13.3 \) and \( \theta = 4^\circ \). Thick lines for OS model and thin lines for two-equation model.
Problem 1

Nonlinear regime: Travelling wave solutions

\[ \xi = (x - ct) \text{ and } \frac{dU}{d\xi} = F(U) \text{, where } U = (h, h', h'')^T \]

\( \gamma_1 \) type

\( \gamma_2 \) type

When \( \Gamma = 3376, \text{ Re } = 2.0667 \) and \( \theta = 90^\circ \).
Problem 1

Periodic wave and Solitary wave

Solitary wave speed

\[ c \rightarrow \delta = \frac{2 \text{Re}}{\kappa v} \]

Periodic wave speed

\[ c \rightarrow \delta = \frac{2 \text{Re}}{\kappa v} \]
DNS(Gerris) : Comparison with model

Maximum amplitude

![Graph showing DNS model and experimental data for maximum amplitude.]

When $\alpha_1 = 0.08$, $We = 76.4$ and $\theta = 90^\circ$.

Speed

![Graph showing DNS model and experimental data for speed.]

When $\alpha_1 = 0.08$, $We = 76.4$ and $\theta = 90^\circ$. 
When $\alpha_1 = 0.08$, $Re = 10$, $We = 76.4$ and $\theta = 90^\circ$. 

sim.avi
DNS : Comparison with model

\begin{align*}
  h(x) \rightarrow
\end{align*}

When $\alpha_1 = 0.08$, $Re = 10$, $We = 76.4$ and $\theta = 90^\circ$.

**Conclusions :**

(i) Slip length provides destabilizing effect near criticality and stabilizing effect far from the criticality.

(ii) Good agreement with Orr-Sommerfeld model in linear regime.

(iii) Good agreement with direct numerical simulations in nonlinear regime.
Falling film over a porous medium: Onedomain approach

Schematic diagram of falling film over a porous medium
- **Introduction**
- Introduce porous medium.
- Consider liquid and porous medium as a composite medium.
- Introduce modified N-S equation.
- Upper and lower boundary are closed by free surface and no slip b.c.
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  • Detail study of linear and nonlinear wave dynamics.
  
  • Influence of permeability on the wave dynamics.
  
  • Study of limiting cases at high and low permeability.
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• **Formulation**
  • Orr-Sommerfeld BVP
  • Base flow distribution.
  • Two-equation model in terms of film thickness $h(x, t)$, flow rate $q(x, t)$.
  • Travelling wave solutions.
  • Time dependent computation.

• **Conclusions**
**Base flow distribution**

Let us consider a uniform flow

\[
\frac{1}{\varepsilon(y)} \partial_y u - \frac{1}{\kappa(y)} u + 1 = 0,
\]

\[
\partial_y u|_1 = u|_0 = 0.
\]

\[
\varepsilon(y) = \frac{(1 + \varepsilon_H)}{2} + \frac{(1 - \varepsilon_H)}{2} \tanh \left( \frac{(y - d/l_\nu)}{\Delta} \right),
\]

\[
\frac{1}{\kappa(y)} = \frac{1}{Da_\nu} \left( \frac{1}{2} - \frac{1}{2} \tanh \left( \frac{(y - d/l_\nu)}{\Delta} \right) \right)
\]

\[
Da_\nu = \kappa_H/l_\nu^2, \text{ Darcy number, } \varepsilon_H, \text{ porosity } \kappa_H, \text{ permeability.}
\]
Momentum diffusion layer thickness

\[ d_B = \sqrt{\frac{\kappa_H}{\epsilon_H}}, \] dimensional diffusion layer thickness, \( l_s \), dimensional slip length, \( d \), dimensional porous layer thickness.

Results:
- Agreement with BJ postulate Beaver and Joseph, JFM (1967).
Reference scale:

$\bar{U}_N$ is the velocity scale, $\bar{H}_N$ is the length scale, $\rho \bar{U}_N^2$ is the pressure scale, $\bar{H}_N/\bar{U}_N$ is the time scale.

Dimensionless numbers:

$Re = \bar{U}_N \bar{H}_N/\nu$, Reynolds number, $We = \sigma/(\rho \bar{U}_N^2 \bar{H}_N)$, Weber number, $Fr^2 = \bar{U}_N^2/(g \bar{H}_N \sin \theta)$, Froude number.

![Diagram of falling film over a porous medium]
Orr-Sommerfeld BVP: Threshold of instability

\[ \frac{Re}{\cot \theta} \rightarrow \frac{dB}{d} \rightarrow 0 \]

\[ \epsilon_H = 0.78, \frac{d}{l_H} = 0.1, \triangle = 0.001, \theta = 4.6 \text{ and } \Gamma = 769.8 \]

Results:
- Recover liquid-liquid limit Yih, Phys Fluids (1963) and Benjamin JFM (1957).
- Recover liquid-solid limit.
BL equations

\[ \partial_x u + \partial_y v = 0, \]
\[ \text{Re} \left[ \partial_t u + \varepsilon^{-1} (u \partial_x u + v \partial_y u) + uv \partial_y \varepsilon^{-1} \right] = \]
\[ \frac{\text{Re}}{\text{Fr}^2} \varepsilon (1 - \cot \theta \partial_x H) + \varepsilon \text{WeRe} \partial_{xxx} H \]
\[ + \mathcal{L} u + \partial_{xx} u + 2\varepsilon \partial_x [\partial_x u|_H] - \varepsilon \partial_x \left[ \varepsilon^{-1} \mathcal{L} v dy \right], \]
\[ u|_0 = v|_0 = 0, \]
\[ \partial_y u|_H = 4\partial_x u|_H \partial_x H - \partial_x v|_H, \]
\[ \partial_t H + u|_H \partial_x H = v|_H, \]
where \( \mathcal{L} \equiv \partial_{yy} - \varepsilon(y)/\kappa(y) \)

\[
u(x, t) = \frac{q(x, t)}{\phi(H)} f(y; H) + u_1,
\]

where \( \phi(H) = \int_0^H f(y; H)\,dy \) and \( \langle u_1|1 \rangle = \int_0^H u_1\,dy = 0 \)

\[
\mathcal{L}f = -\varepsilon(y), \quad f(0; H) = \partial_y f(H; H) = 0,
\]

weight defined by \( \mathcal{L}w = \text{cst} \), i.e.

\[
\mathcal{L}w = -\frac{\text{Re}}{\text{Fr}^2} = -\frac{1}{f(1; 1)}, \quad w(0; H) = \partial_y w(H; H) = 0,
\]

\[
g = \partial_H f \equiv \lim_{\delta H \to 0} f(y; H + \delta H) - f(y; H)/\delta H \text{ solution to}
\]

\[
\mathcal{L}g = 0, \quad g(0; H) = 0 \quad \text{and} \quad \partial_y g(H; H) = 1,
\]
\[
F(H) = \frac{1}{\phi^2 H} \int_0^H \left[ \varepsilon' f l + \varepsilon (f^2 - f'l) + \varepsilon^2 (\phi' f - \phi g) \right] w/\varepsilon^2 dy,
\]
\[
G(H) = \frac{1}{\phi^3} \int_0^H \left\{ \varepsilon' f (\phi' l - \phi m) + \varepsilon \left[ \phi' (f^2 - f'l) + \phi (f'm - fg) \right] \right\} w/\varepsilon^2 dy,
\]
\[
S(H) = \frac{1}{\phi H^2} \int_0^H f w \, dy, \quad I(H) = \Upsilon/H^2,
\]
\[ J(H) = \frac{1}{\phi^3} \left\{ \right. \\
\left. \gamma \left[ \phi^2 + 4f|_H (\phi')^2 - 2\phi(f|_H + \phi')\phi'' \right] + w|_H \phi(\phi'' - 2f|_H \phi'') + \int_0^H 2(\phi')^2 f - 2\phi\phi' g - \phi\phi'' f \\
+ \varepsilon \left[ 2(\phi')^2 - \phi\phi'' \right] (n - r) + 2\phi\phi'(s - o) \right\} w \, dy, \]

\[ K(H) = \frac{2}{H^3 \phi^2} \left\{ \right. \\
\left. 2f|_H \gamma \phi' - \gamma \phi\phi'' - f|_H w|_H \phi \\
+ \int_0^H \phi' f - \phi g + \varepsilon (\phi'(n - r) + \phi(s - o)) \right\} w \, dy, \]

\[ L(H) = \frac{H^2}{2} K(H) + 2 \frac{f|_H w|_H}{\phi}, \]

\[ M(H) = \frac{1}{H^2 \phi} \left\{ \right. \\
\left. 2f|_H \gamma + w|_H \phi + \int_0^H [f + \varepsilon (n - r)] w \, dy \right\}, \]
where

\[
\begin{align*}
g(y; H) &= \partial_H f, \quad l(y; H) = \int_0^y f \, dy, \\
m(y; H) &= \int_0^y g \, dy, \quad n(y; H) = \int_H^y \varepsilon^{-1} \partial_y f \, dy, \\
o(y; H) &= \int_H^y \varepsilon^{-1} \partial_y g \, dy, \quad r(y; H) = \int_H^y l/\kappa \, dy, \\
s(y; H) &= \int_H^y m/\kappa \, dy, \quad \phi(H) = \int_0^H f \, dy, \quad \text{and} \quad \Upsilon(H) = \int_0^H \varepsilon w \, dy.
\end{align*}
\]
Two-equation model:

\[ \partial_t H + \partial_x q = 0, \]

\[ \text{Re} \left( S(H)\partial_t q + F(H)\frac{q}{H}\partial_x q - G(H)\frac{q^2}{H^2}\partial_x H \right) = - \frac{\text{Re} q}{\text{Fr}^2 \frac{H^2}{H}} \]

**Inertia term**

\[ + I(H)\frac{\text{Re}}{\text{Fr}^2} (H - \cot \theta H \partial_x H) + I(H)\text{WeReH}\partial_{xxx} H \]

**Capillary term**

\[ + \left( J(H)\frac{q}{H^2}(\partial_x H)^2 + K(H)\frac{\partial_x q\partial_x H}{H} + M(H)\partial_{xx} q + L(H)\frac{q}{H}\partial_{xx} H \right) \]

**Streamwise viscous diffusion term**

\[ S(H) = \frac{1}{\phi H^2} \int_0^H \text{fwdy}, \quad \phi(H) = \int_0^H fdy \quad (1) \]
Two-equation model: Comparison with OS

\[ \varepsilon_H = 0.78, \, d/l_\nu = 0.1, \, Da_\nu = 0.1, \, \Delta = 0.001, \, \theta = 4.6 \text{ and } \Gamma = 769.8. \]

Solid, dashed and dotted lines corresponding to OS, second order model and first order model respectively.
Two-equation model : Marginal curve

$$\varepsilon_H = 0.78, \frac{d}{l_\nu} = 0.1, \triangle = 0.001, \theta = 4.6 \text{ and } \Gamma = 769.8$$
Marginal curves: Moving liquid layer

\[ \varepsilon_H = 0.78, \; d/l_\nu = 0.1, \; \Delta = 0.001, \; \theta = 4.6 \text{ and } \Gamma = 769.8. \]
\[
\begin{align*}
\text{Da}_ν &= 0.1 \\
\text{Da}_ν &= 0.01 \\
\text{Da}_ν &= 0.001
\end{align*}
\]
Travelling wave solutions : Wave characteristics

Maximum amplitude

\[ h_{\text{max}} \rightarrow \]

\[ \Lambda \rightarrow \]

\[ Da_\nu = 1 \]
\[ Da_\nu = 0.1 \]
\[ Da_\nu = 0.01 \]
\[ Da_\nu = 0.001 \]
\[ Da_\nu = 0.00001 \]

Speed

\[ c \rightarrow \]

\[ \Lambda \rightarrow \]

\[ Da_\nu = 1 \]
\[ Da_\nu = 0.1 \]
\[ Da_\nu = 0.01 \]
\[ Da_\nu = 0.001 \]
\[ Da_\nu = 0.00001 \]

\[ \varepsilon_H = 0.78, \ d/l_\nu = 0.1, \ \Delta = 0.001, \ \Gamma = 769.8 \] and \[ \Lambda = \text{Re}U(1)/\kappa_\nu, \] modified Froude number

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Maximum amplitude : Moving liquid layer

\[
\tilde{h}_{\text{max}} \rightarrow \Lambda \rightarrow \\
\begin{align*}
Da_\nu &= 0.00001 & \text{red} \\
Da_\nu &= 0.001 & \text{green dash} \\
Da_\nu &= 0.01 & \text{blue dot} \\
Da_\nu &= 0.1 & \text{magenta dot} 
\end{align*}
\]
Travelling wave solutions: Periodic solution

Wave profile

Flow rate

$h(x) \rightarrow$

$q(x) \rightarrow$

$\epsilon_H = 0.78, d/l_\nu = 0.1, \triangle = 0.001, \Gamma = 332$ and $Re = 10.$
Travelling wave solutions: Streamline distribution

Recirculation pattern

Back flow phenomenon under liquid medium

Back flow phenomenon under porous medium

\[ \varepsilon_H = 0.78, \quad d/l_H = 0.1, \quad \triangle = 0.001, \quad \Gamma = 332, \quad Da_\nu = 0.1 \text{ and } Re = 10. \]
Problem II

Time dependent computations:

Periodic external forcing (f=7Hz)

Random forcing

Movie

Movie

\[ \epsilon_H = 0.78, \frac{d}{l_\nu} = 0.1, \Delta = 0.001, \Gamma = 332, Da_\nu = 0.1 \text{ and } Re = 10. \]
Time dependent computations: Spatiotemporal diagram

Periodic external forcing (f=7Hz)

Random forcing

\[ \varepsilon_H = 0.78, \frac{d}{l_\nu} = 0.1, \Delta = 0.001, \Gamma = 332 \text{ and } Re = 10. \] Vertical and horizontal axes are respectively the x axis and t axis.
Liu and Gollub’s experiment
Liu and Gollub’s experiment

\[ \text{Da}_\nu = 10^{-5} \ (d_B = 0.037 \text{ mm}) \]

\[ \text{Da}_\nu = 1 \ (d_B = d = 0.14 \text{ mm}) \]
Problem II

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Conclusions:

- Permeability shows destabilizing effect near criticality and stabilizing effect far from the criticality.
- At low and high permeability, system behaves like a liquid-solid and liquid-liquid limit.
- Good agreement with Orr-Sommerfeld model in linear regime.
- The free surface wave dynamics are completely depends on the moving liquid layer.
- Recirculation phenomenon appear at the core region of the waves.
- Regular structure of localized pulses appear at periodic external forcing and randomly distributed localized pulses appear at external noise.