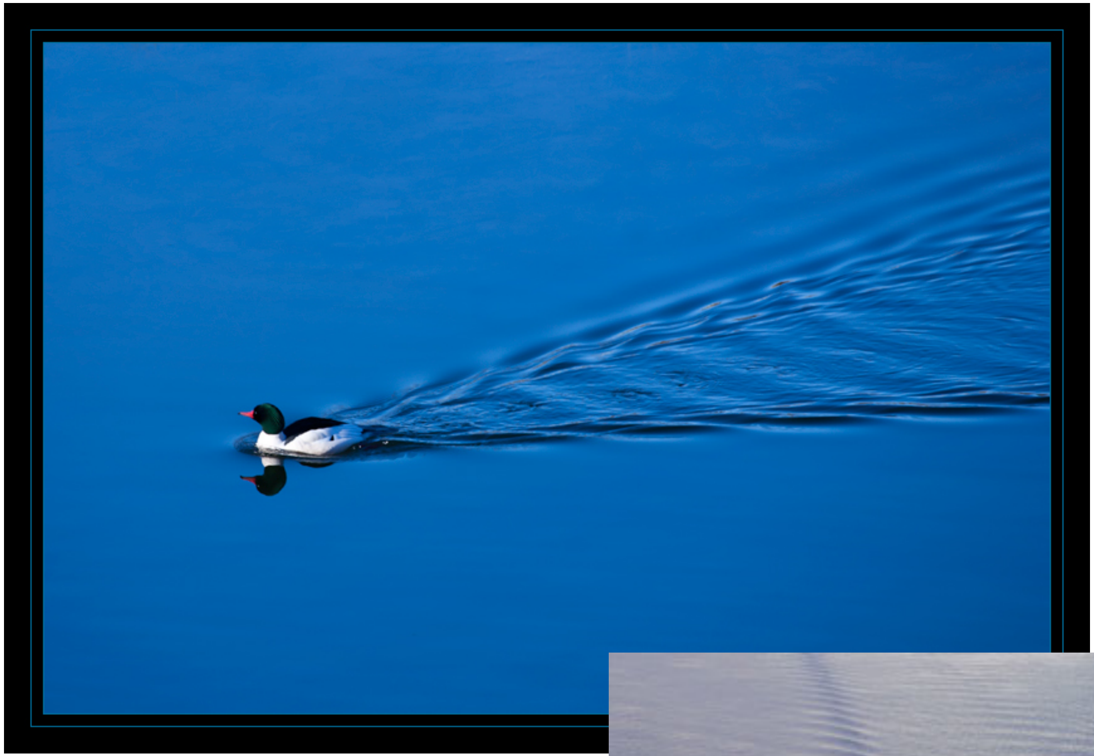
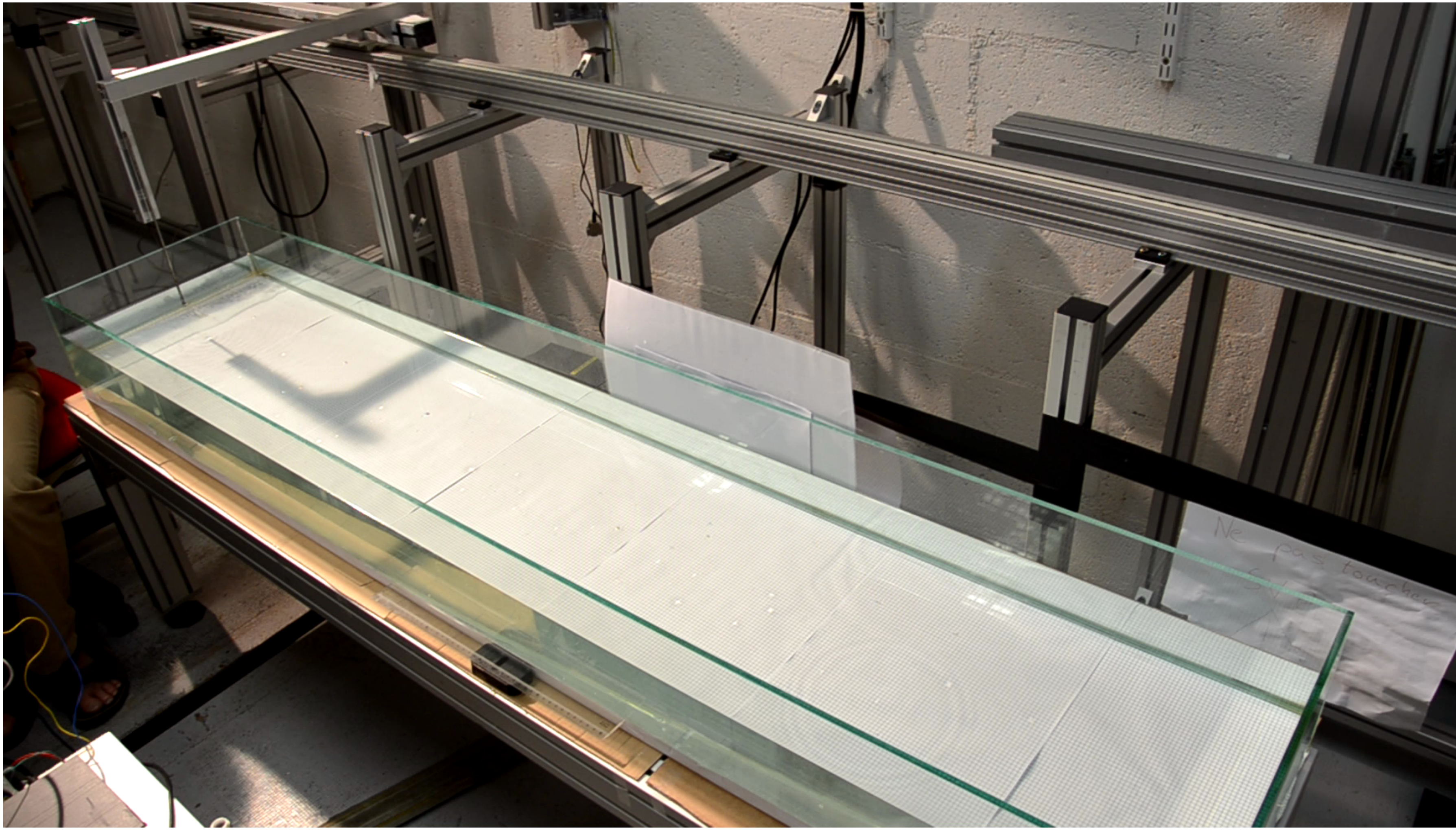
Narrow Kelvin Waves

Frédéric Moisy, Marc Rabaud

GDR PHENIX, Paris, September 19, 2013



Google images


Ne pas toucher

Summary

1) Classical Kelvin wedge for gravity waves
2) Narrow Kelvin wedge at large Froude
3) Effect of capillarity
4) Wave drag and planing hull
5) Conclusions

Dispersion relation

Monochromatic plane waves:

=> Wave equation

$$A = A_0 \exp i(kx - \omega t)$$

Hypothesis: irrotational flow (low dissipation) and linearity (small amplitude)

Dispersion relation $\omega = f(k)$:

$$\omega^2 = gk\left[1 + \left(\frac{k}{k_c}\right)^2\right] \qquad k_c = \sqrt{\frac{\rho g}{\gamma}}$$

Phase velocity:

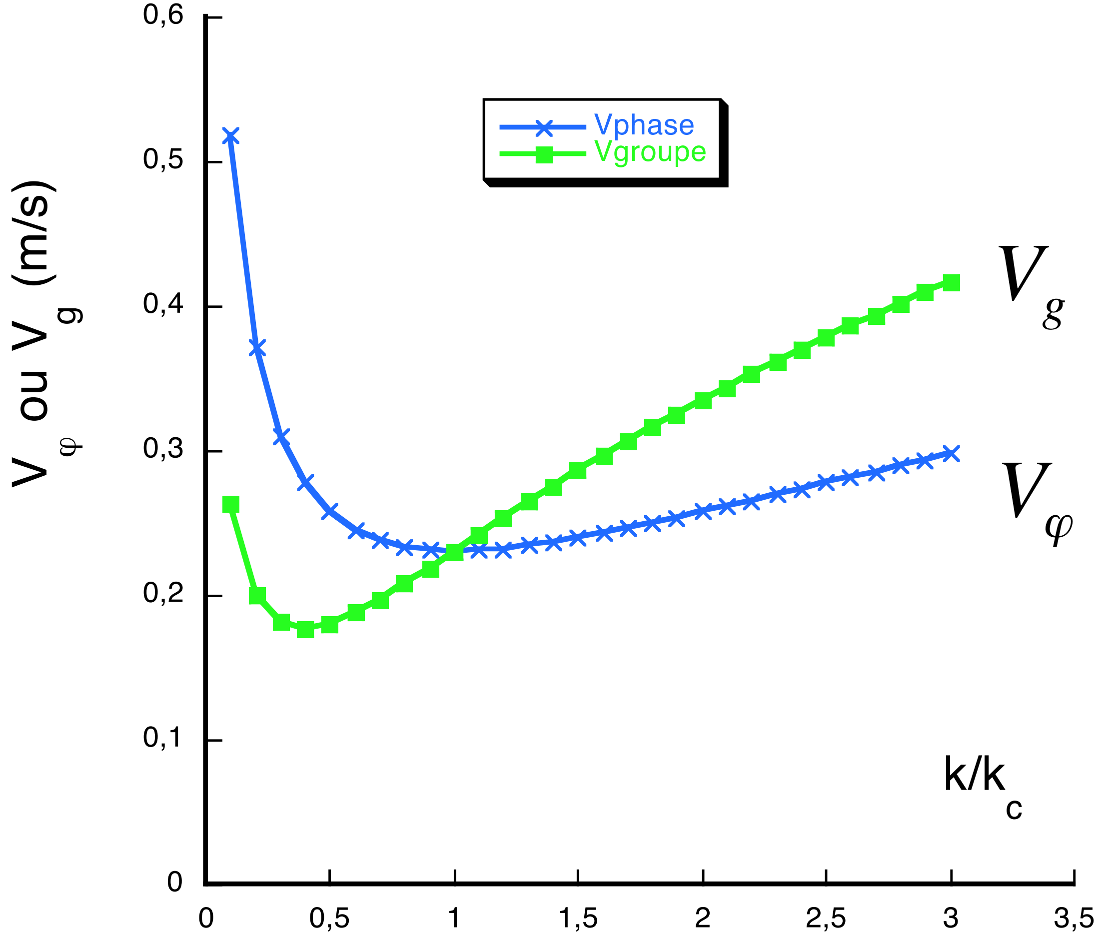
$$V_\varphi(k) = \frac{\omega}{k} = \sqrt{\frac{g}{k}\left[1 + \left(\frac{k}{k_c}\right)^2\right]}$$

Dispersive waves

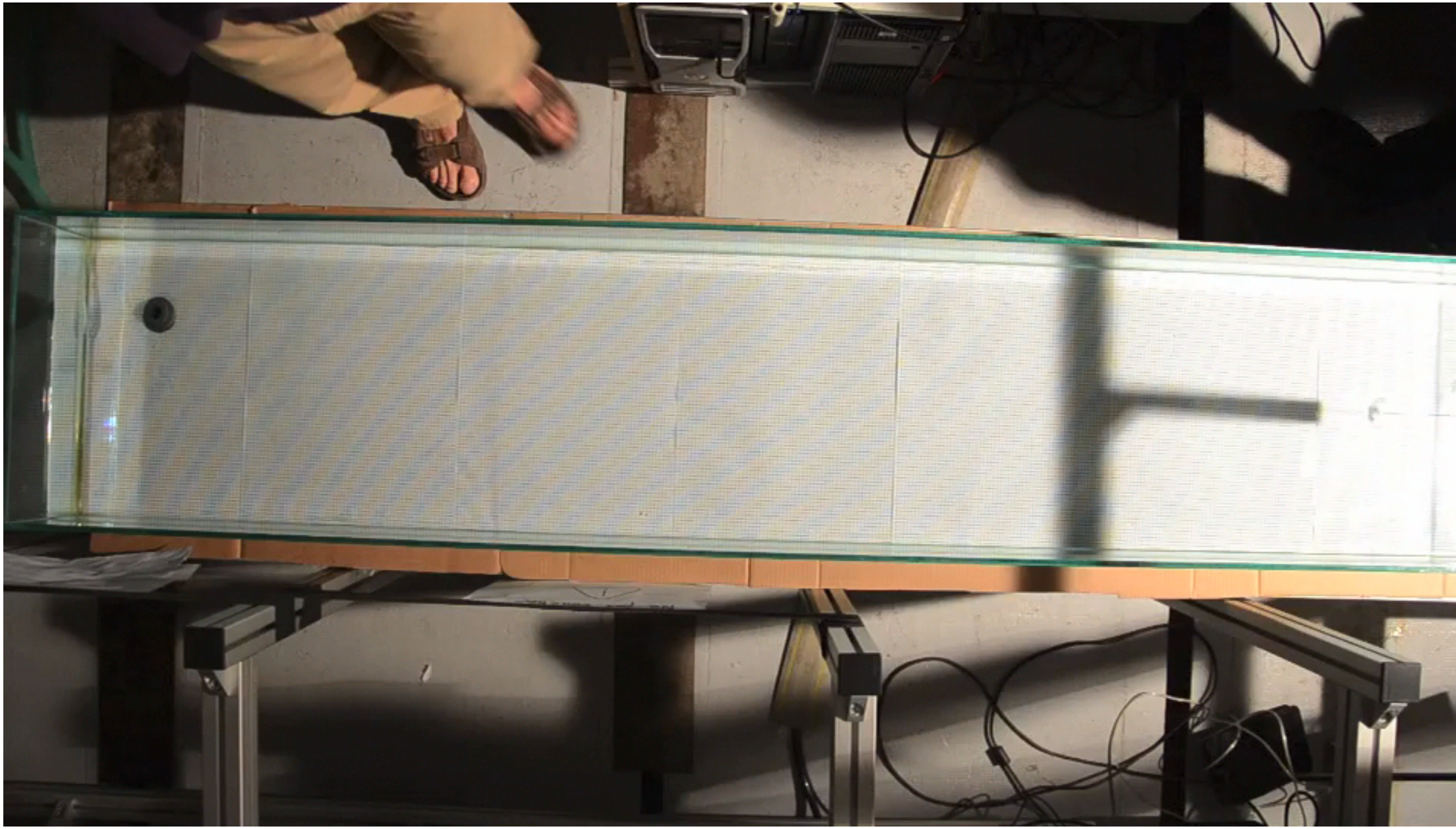
Group velocity:

$$V_g(k) = \frac{d\omega}{dk}$$

Phase and group velocities in deep water

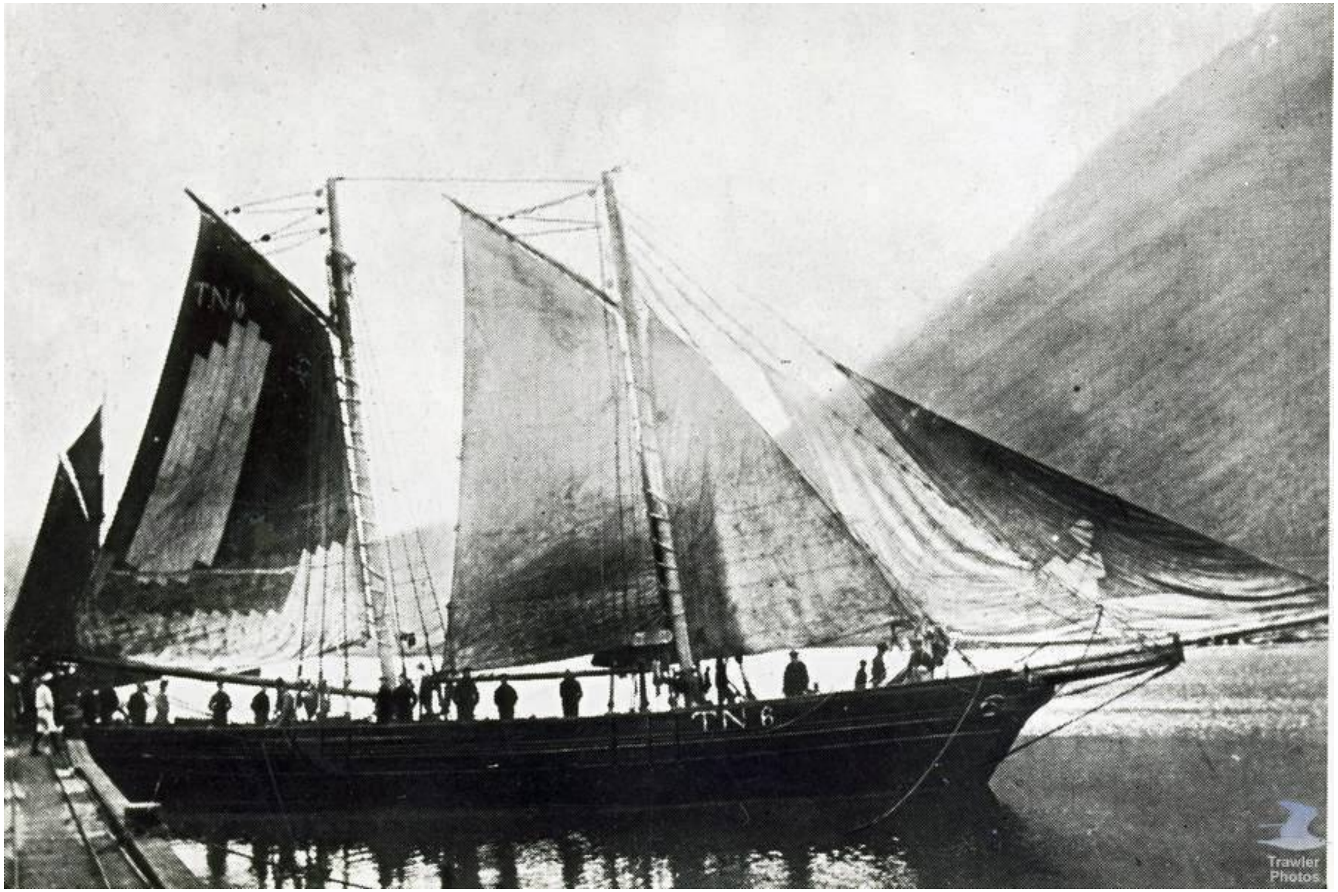


Close to minimum velocity



1,5 mm cylinder moving at 22 cm/s

Existence of V_{min}: Thomson verified this point on his yacht with the help of an eminent guest, Hermann von Helmholtz [O. Darrigol, *Worlds of flow*].



Kelvin's floating laboratory: the Lalla Rookh

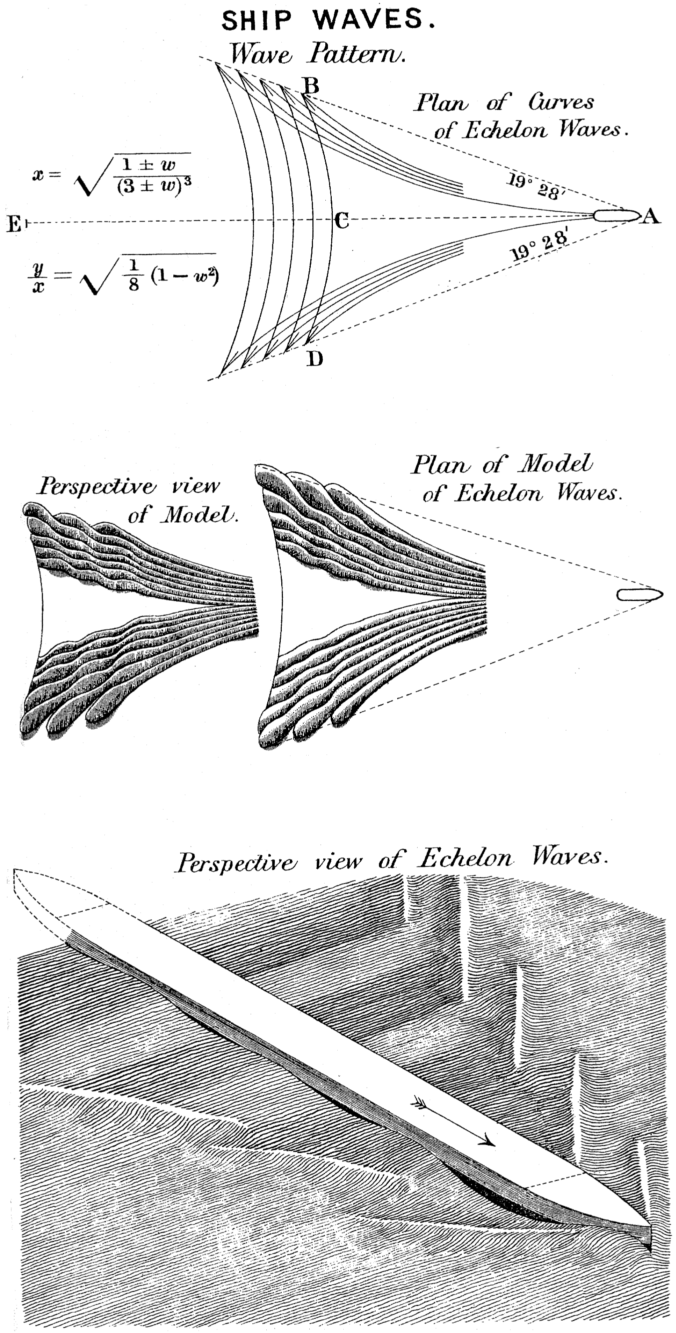
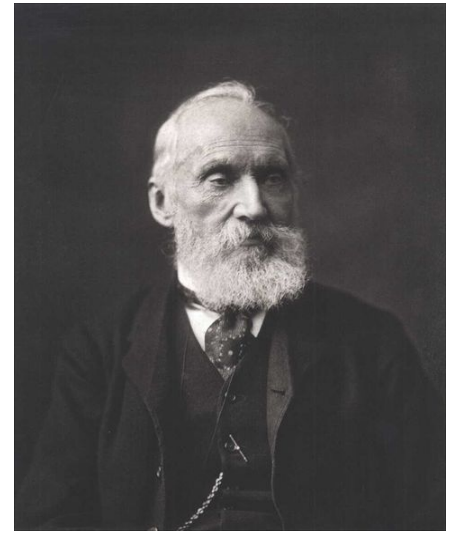


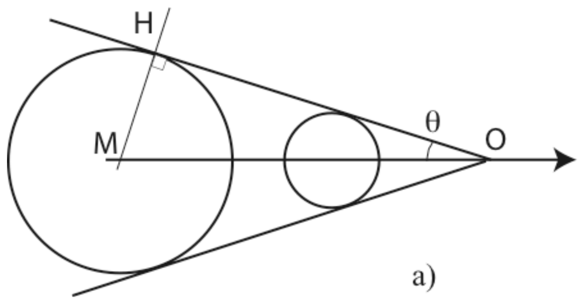
Fig. 25. Kelvin's ship-waves [Thomson 1887a, plate; perspective view borrowed by Kelvin from R.E. Froude]



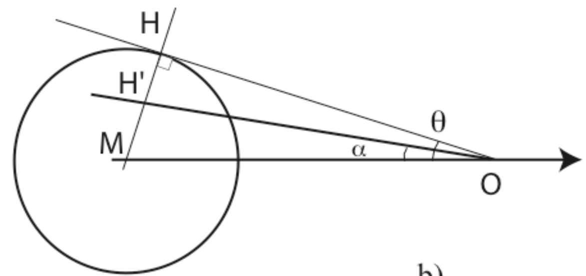
Lord Kelvin
(William Thomson)
1824-1907

William Froude
(1810-1878)

Mach construction or Kelvin wedge?



a)



b)

Mach cone (U > c)

Non dispersive media

$$C = V_g = V_\varphi$$

$$\sin\alpha(U) = \sin\theta = \frac{C}{U}$$

Kelvin wedge (any U)

Dispersive media $V_g = \frac{1}{2} V_\varphi$

$$U \sin\theta(k) = V_\varphi = \sqrt{\frac{g}{k}}$$

$$\tan\alpha = \frac{\tan\theta}{2 + \tan^2\theta}$$

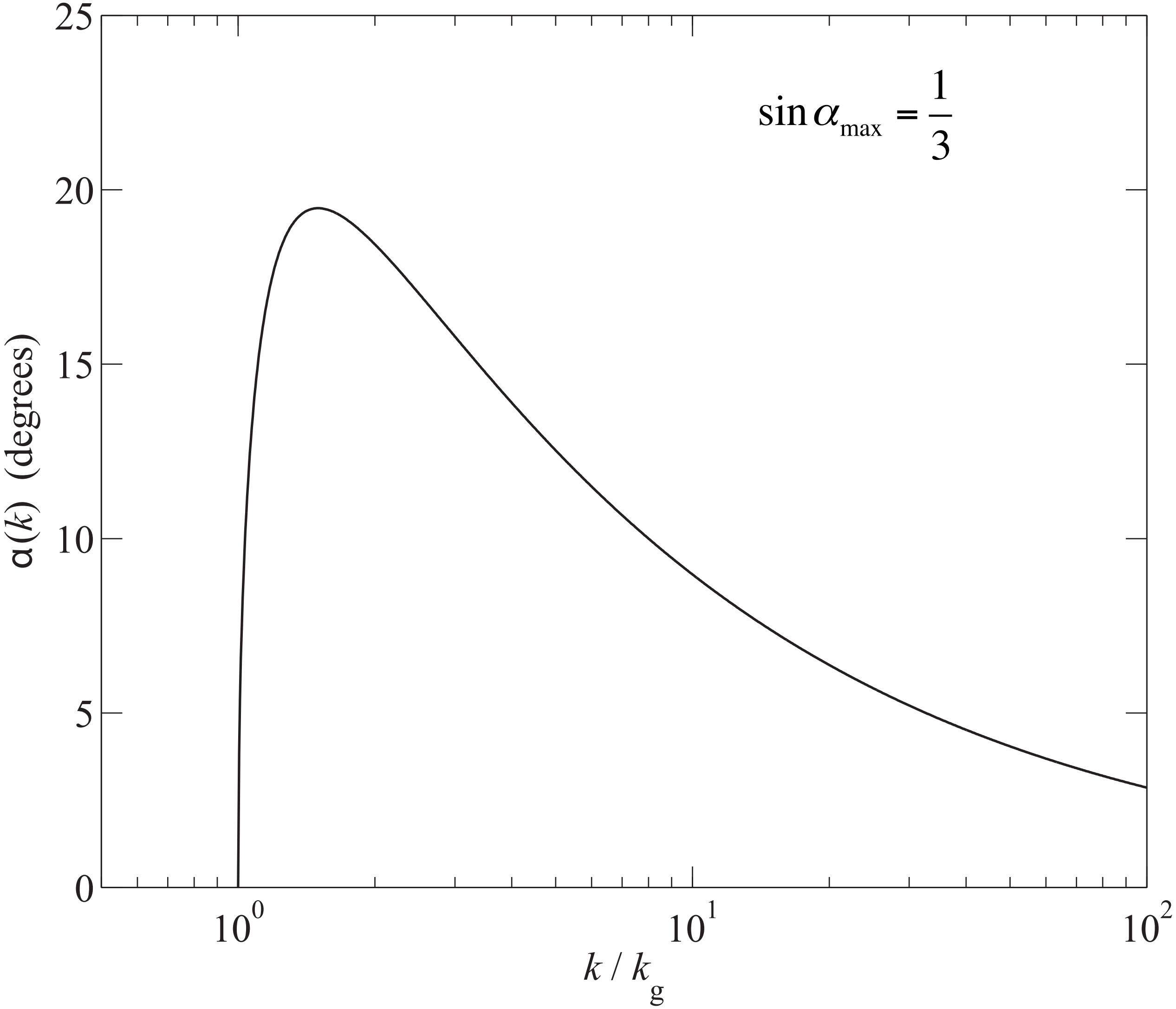
- *Crawford, AJP, 1984*
- *Lamb, Lighthill or Whitham*

$$\tan\alpha(\theta) = \frac{\tan\theta}{2+\tan^2\theta}$$

$$U\sin\theta(k) = V_\varphi = \sqrt{\frac{g}{k}}$$

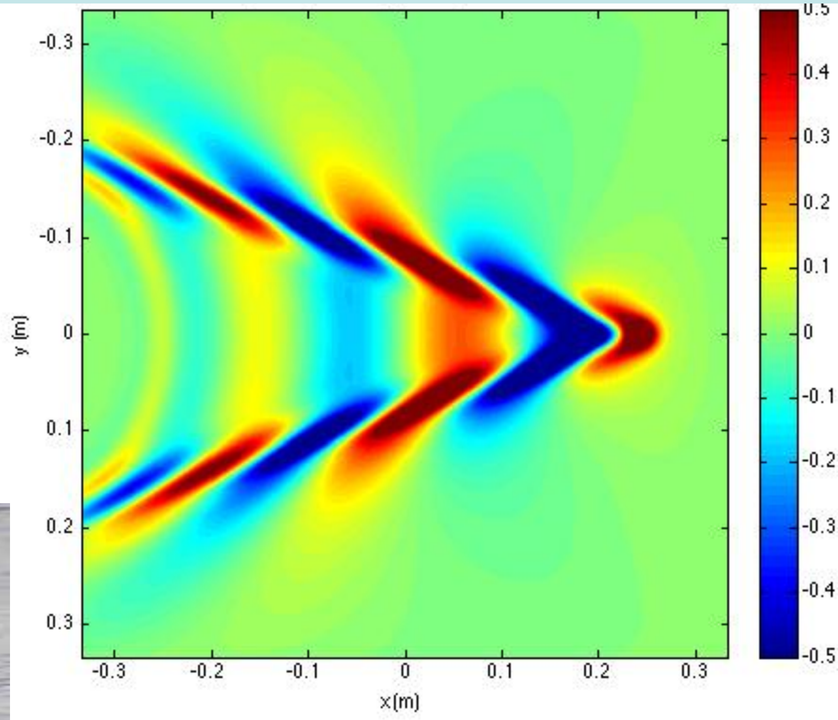
$$\tan\alpha(k) = \frac{\sqrt{\dfrac{k}{k_g}-1}}{2\dfrac{k}{k_g}-1}$$

avec $k_g = \dfrac{g}{U^2}$



Everything clear?

Photograph by **Adrian Pingstone** (Avon Gorge, Bristol, 2004)



Matlab simulation (F. Moisy)

Summary

1) Classical Kelvin wedge for gravity waves
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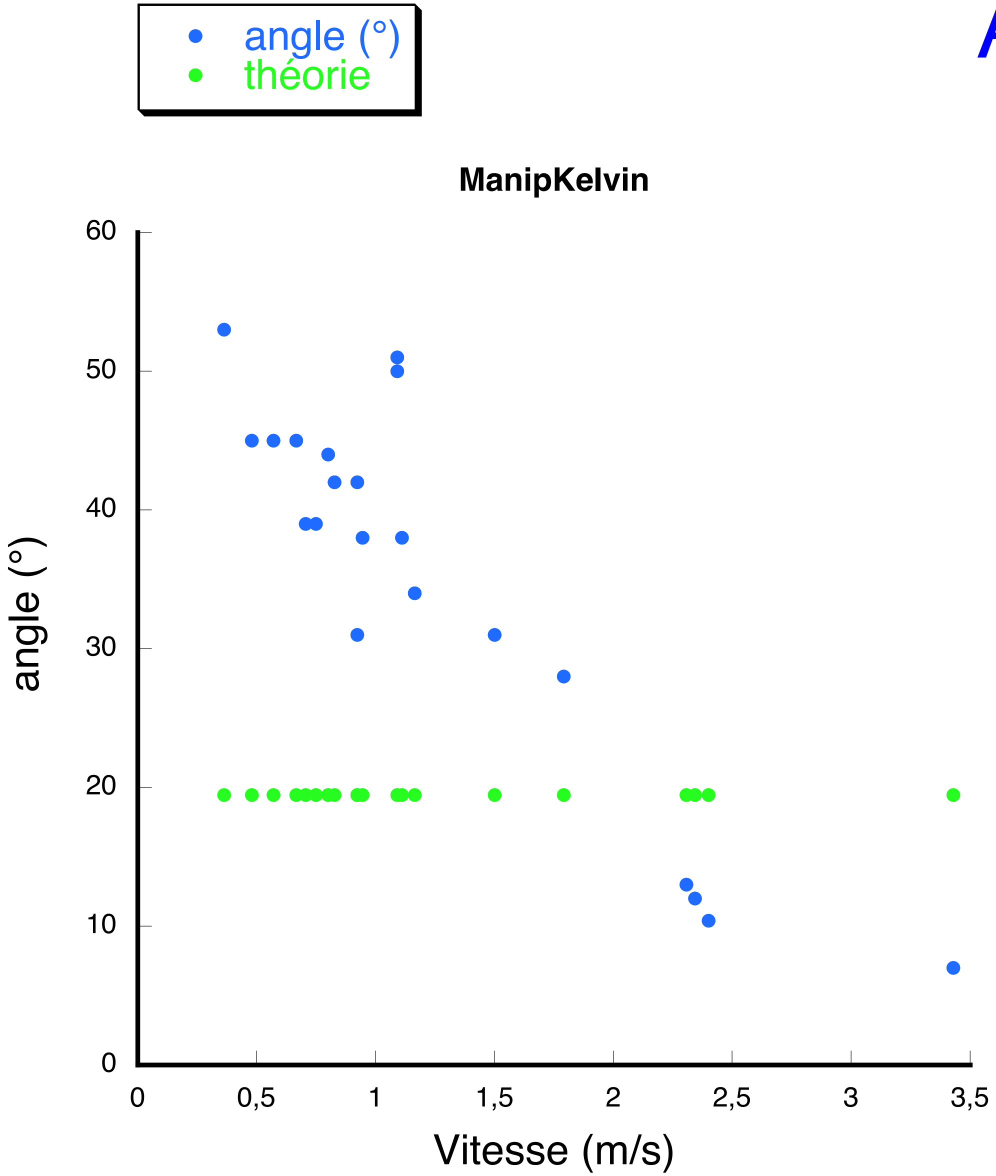
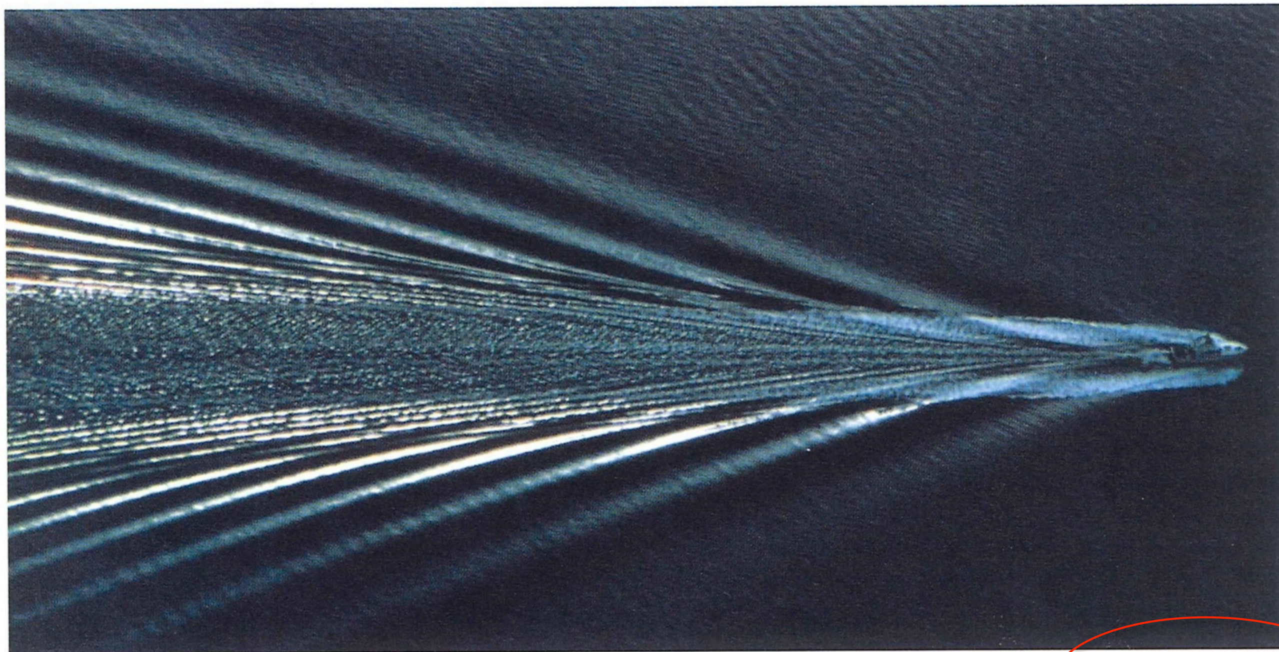
5) Conclusions

And yet …

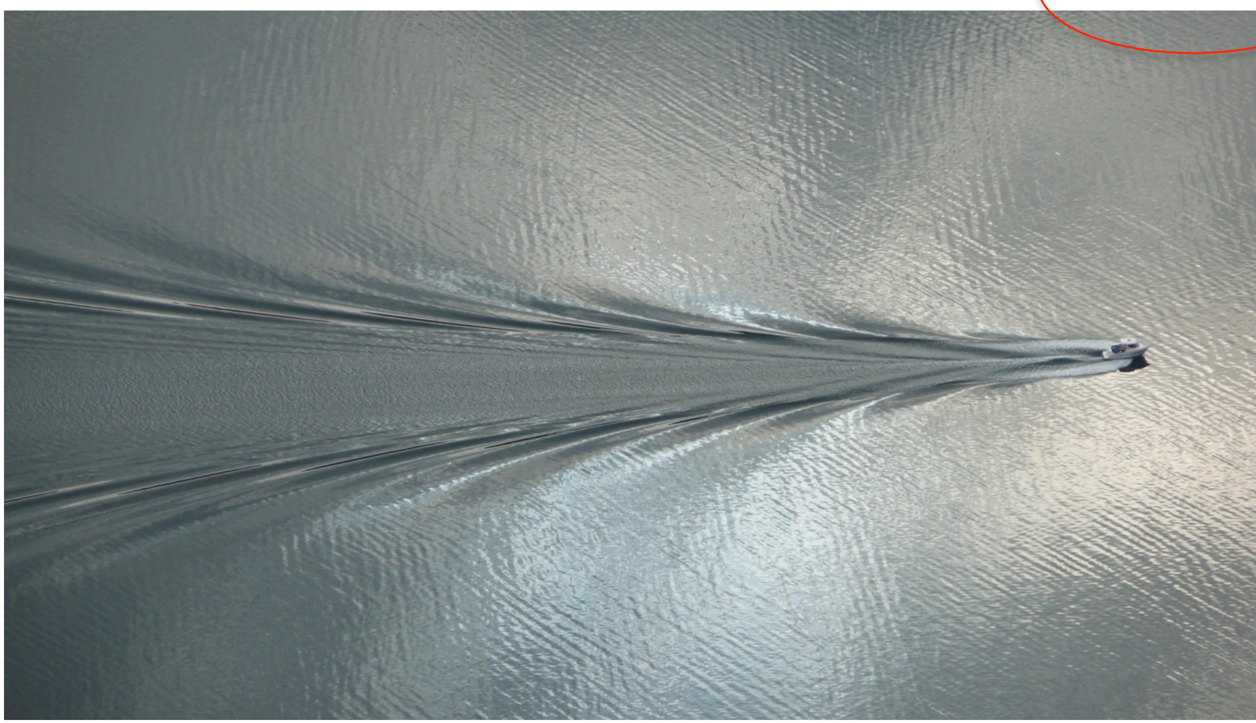


Les Treilles
(octobre 2001)

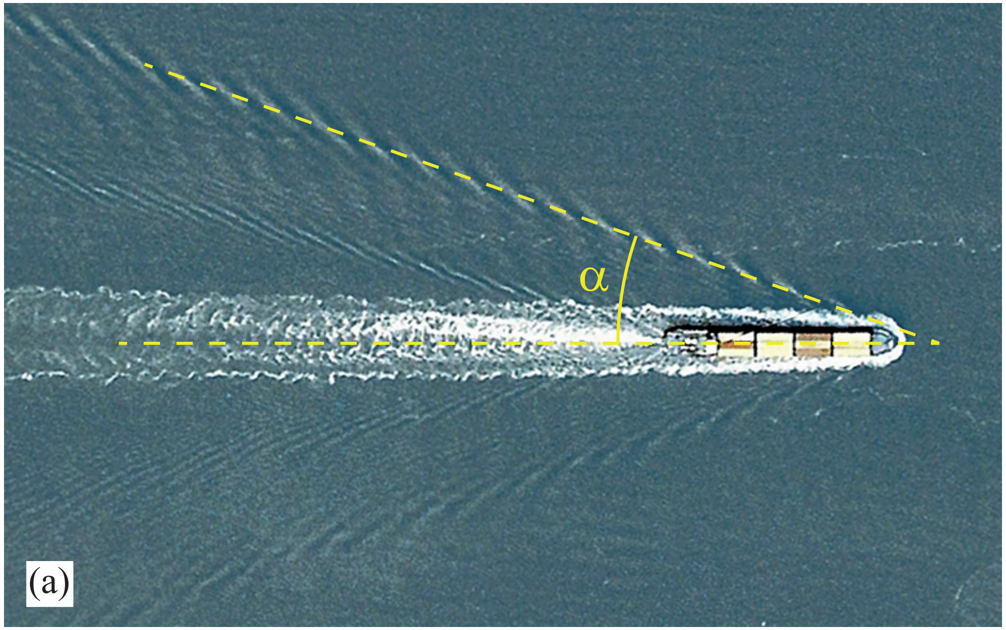


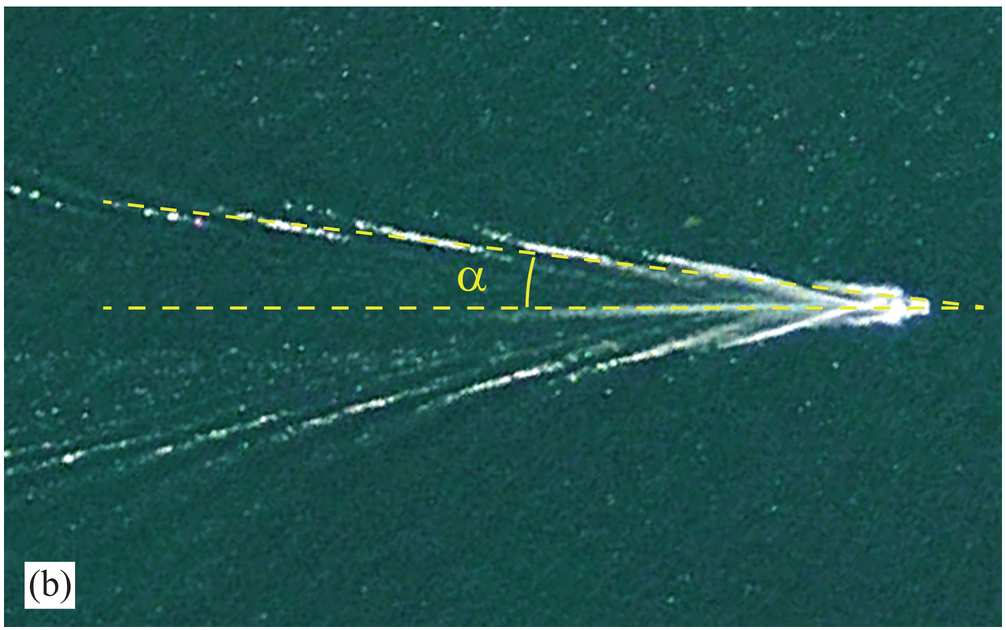
1. VUE AÉRIENNE du sillage d'un bateau à moteur. La figure d'interférence est limitée par un *V* faisant un angle de 39 degrés.

Jear Walker
(Pour la Science)



Wikipedia





« Field measurements »

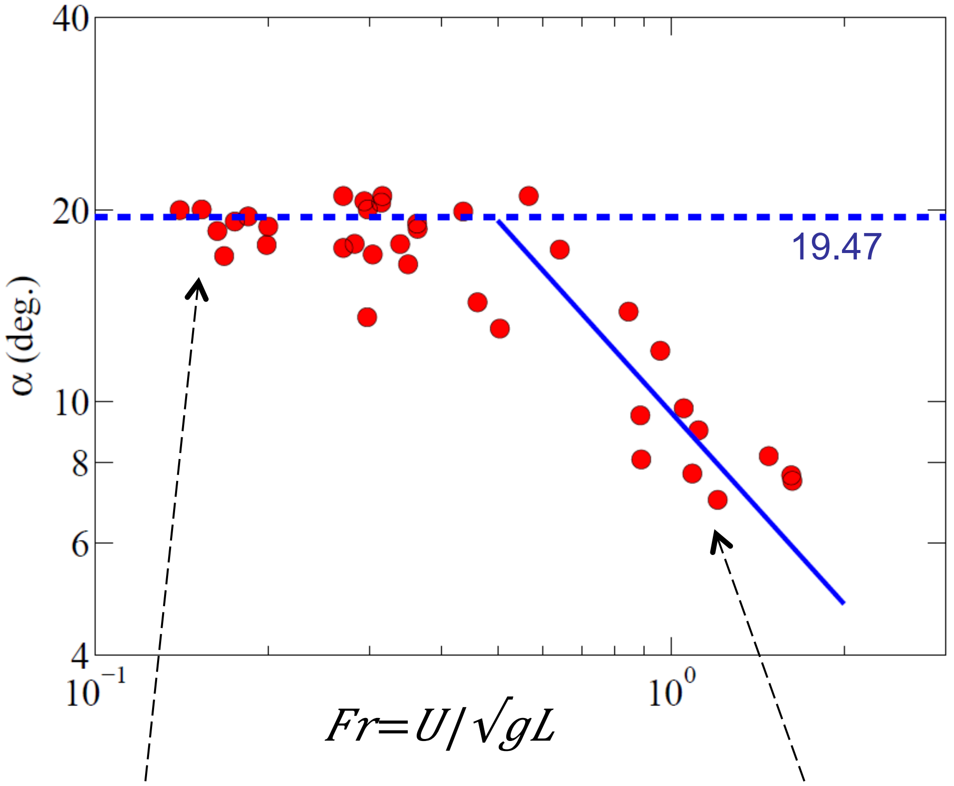
Set of 37 images from *Google Earth* close to harbors (resolution 0.5 m)

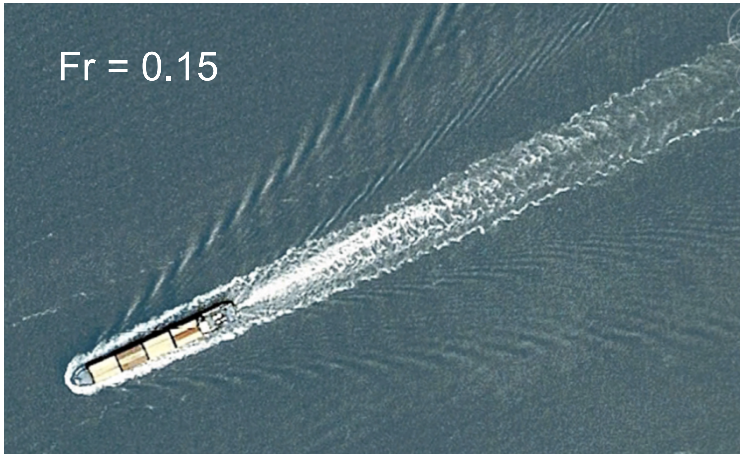
Measure ship length *L* and wavelength λ

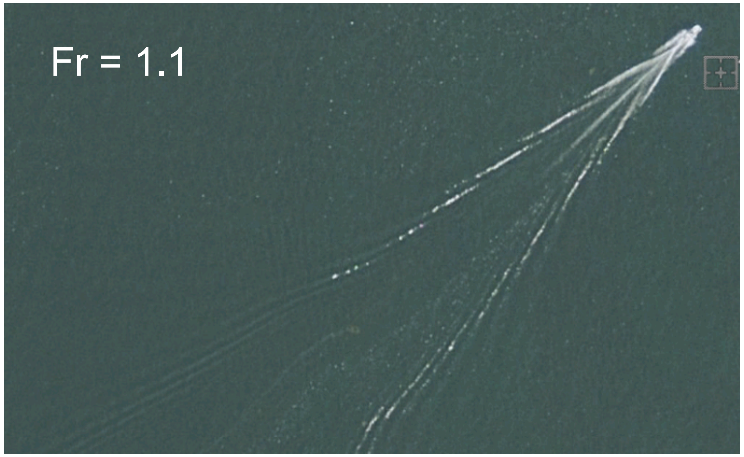
Deduce ship velocity *U* and Froude number

$$Fr = \frac{U}{\sqrt{gL}} \qquad \alpha = f(Fr)?$$

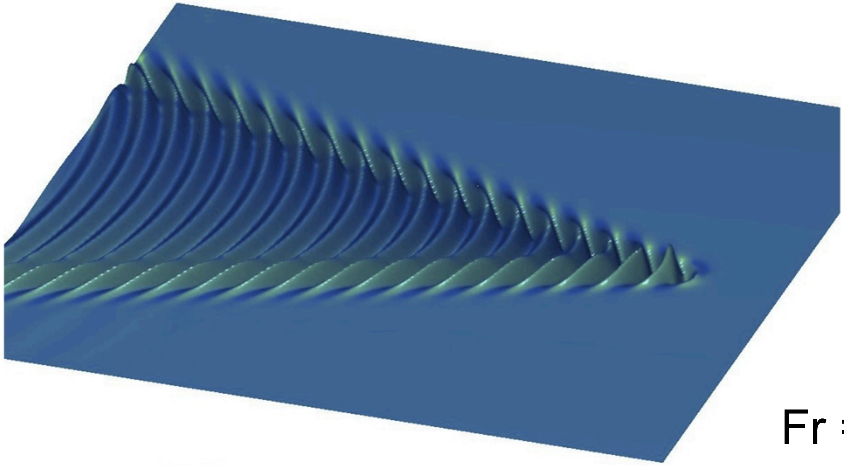
Wake angles from Google Earth images



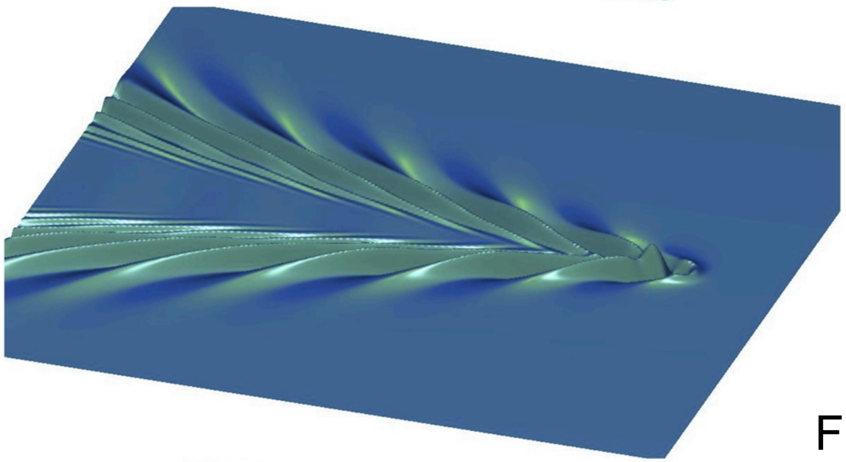




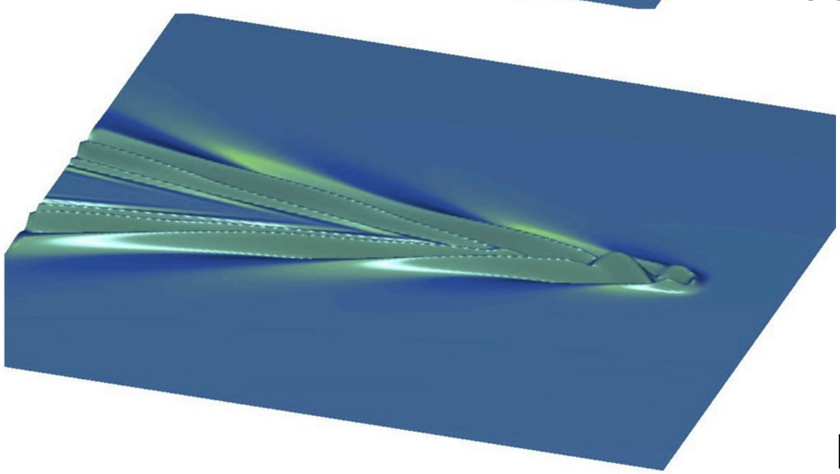
Simulation with gaussian
pressure field
(Havelock hypothesis 1918)



Fr =0.5

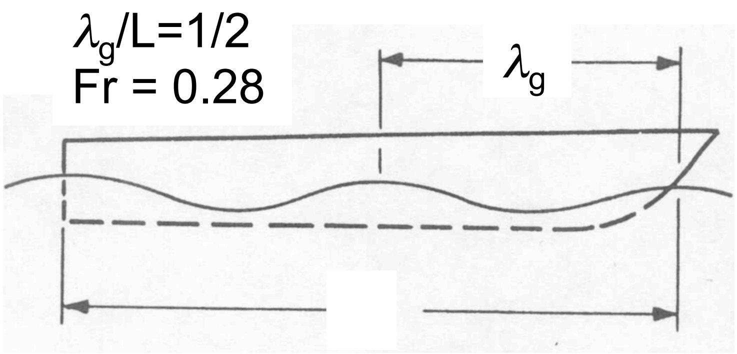


Fr =1



Fr =2

Origin of narrow wake pattern ? Finite size effect



Largest excited gravity wave:

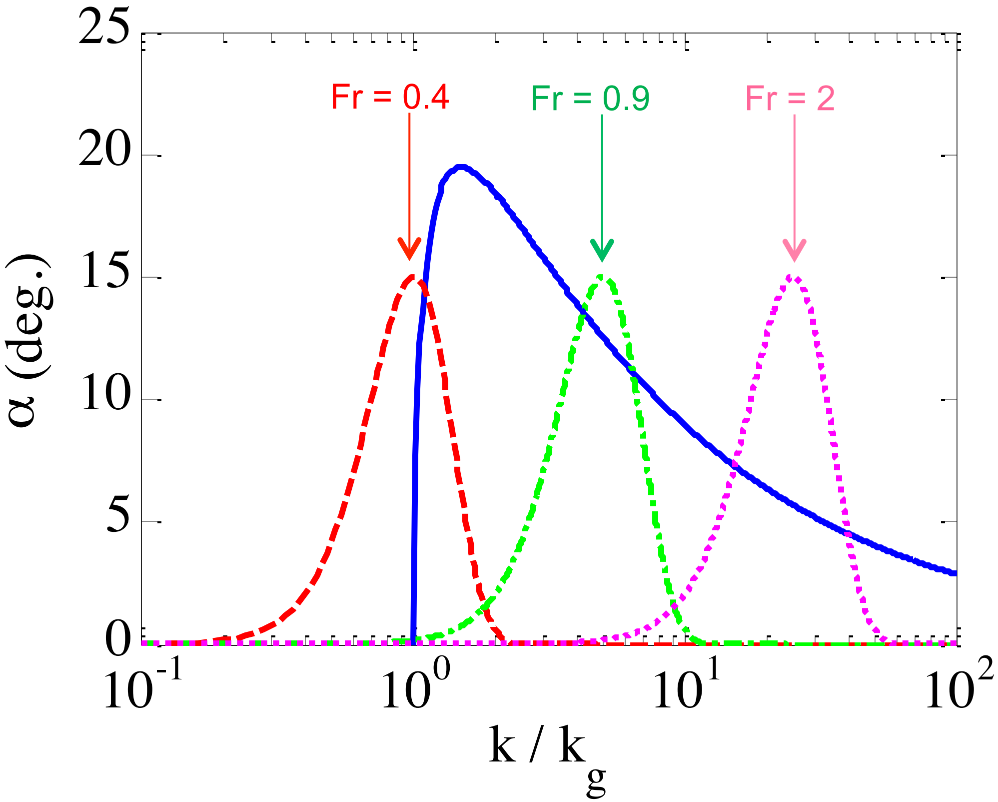
$$\lambda_g = 2\pi/k_g = 2\pi U^2/g$$

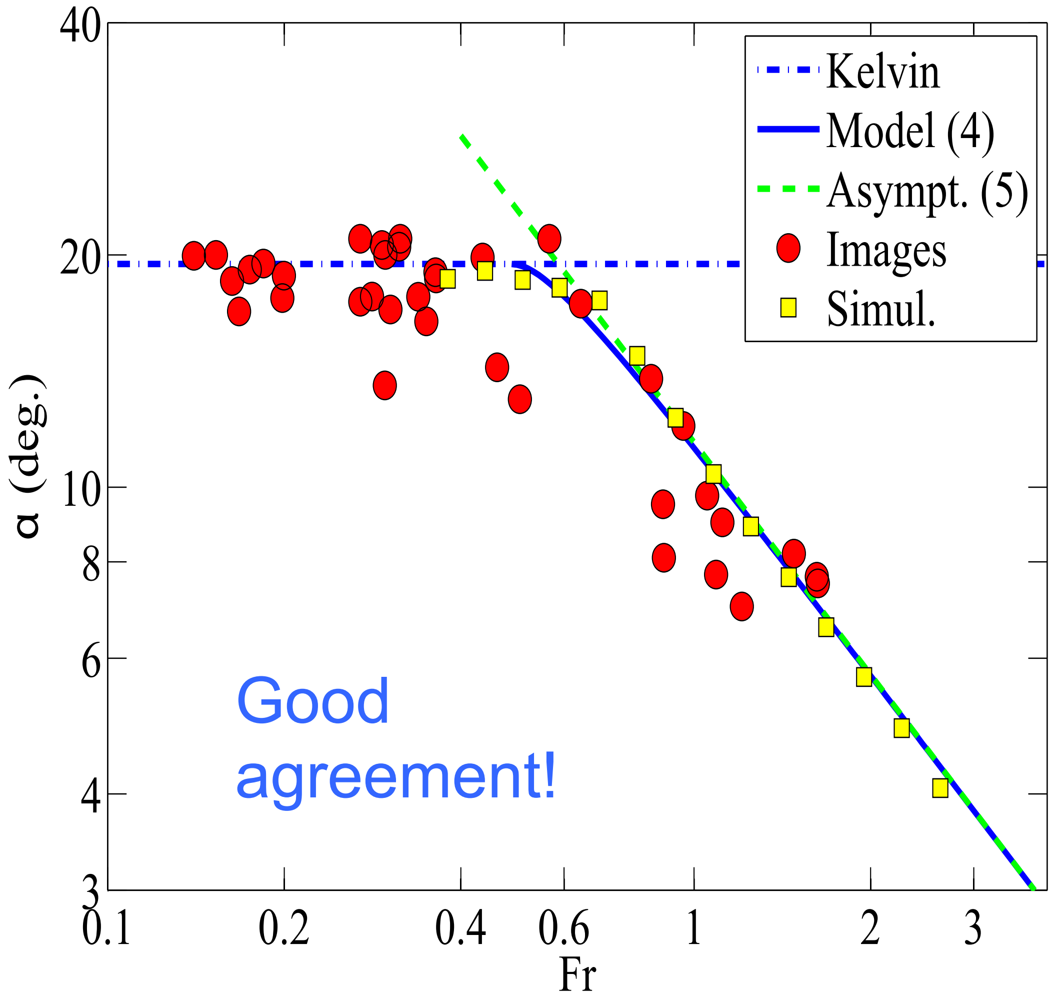
$$Fr = \sqrt{\lambda_g/2\pi L}$$

The actual wake pattern is an convolution of

$$\tan\alpha(k) = \frac{\sqrt{k/k_g - 1}}{2k/k_g - 1}$$

by the spectrum of the disturbance





- 2 lengths L and λ_g,
- 1 dimensionless number Fr

At large Froude:

$$\sin(\alpha) \sim \frac{1}{Fr} = \frac{\sqrt{gL}}{U}$$

As in the Mach regime where $$\sin(\alpha) = \frac{c}{U}$$

Phys. Rev. Letters 110, 214503 (mai 2013)

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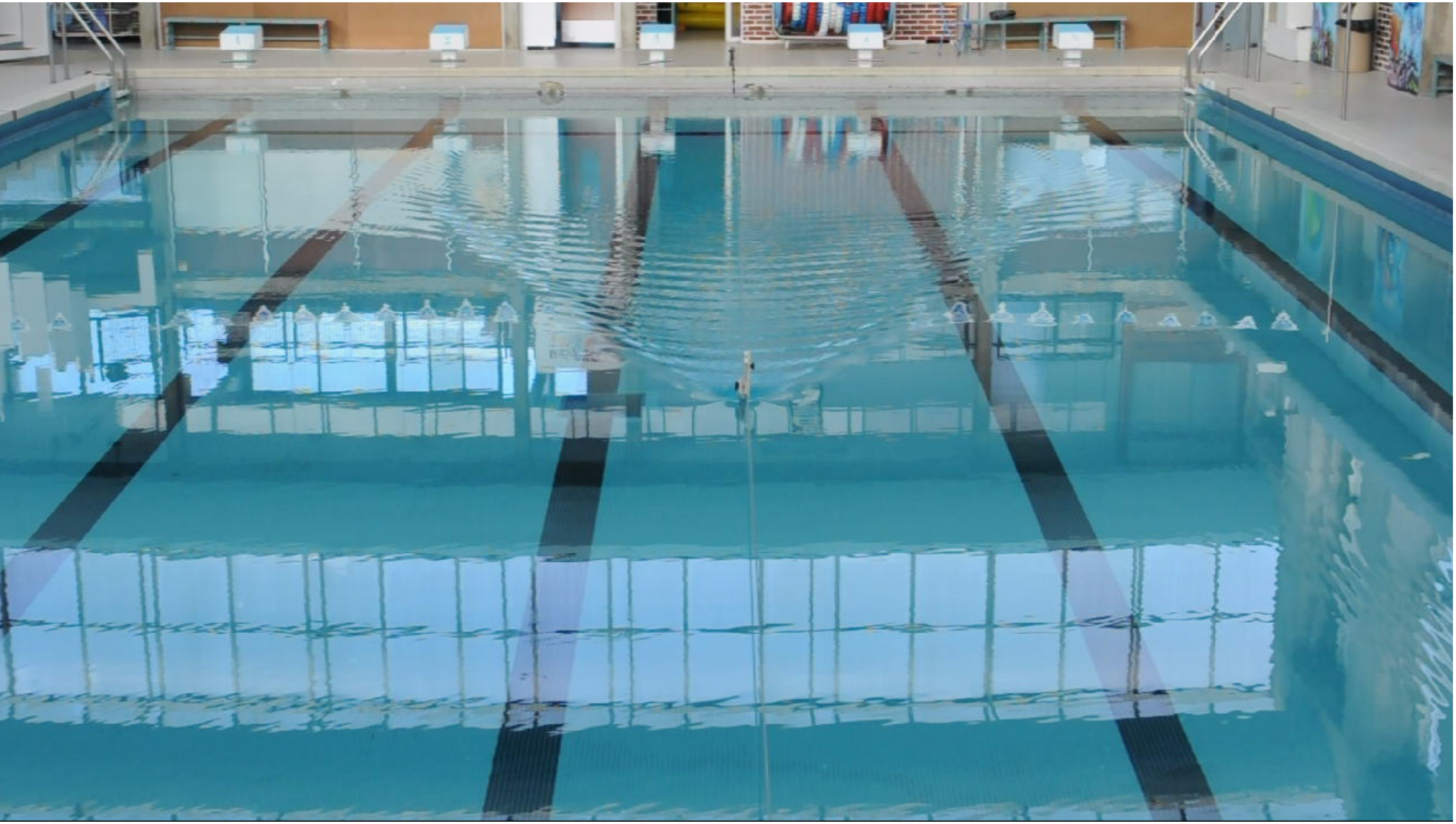
‘Cylindrical duck’ of diameter 3 cm, U = 0.75 m/s (Fr = 1.4)

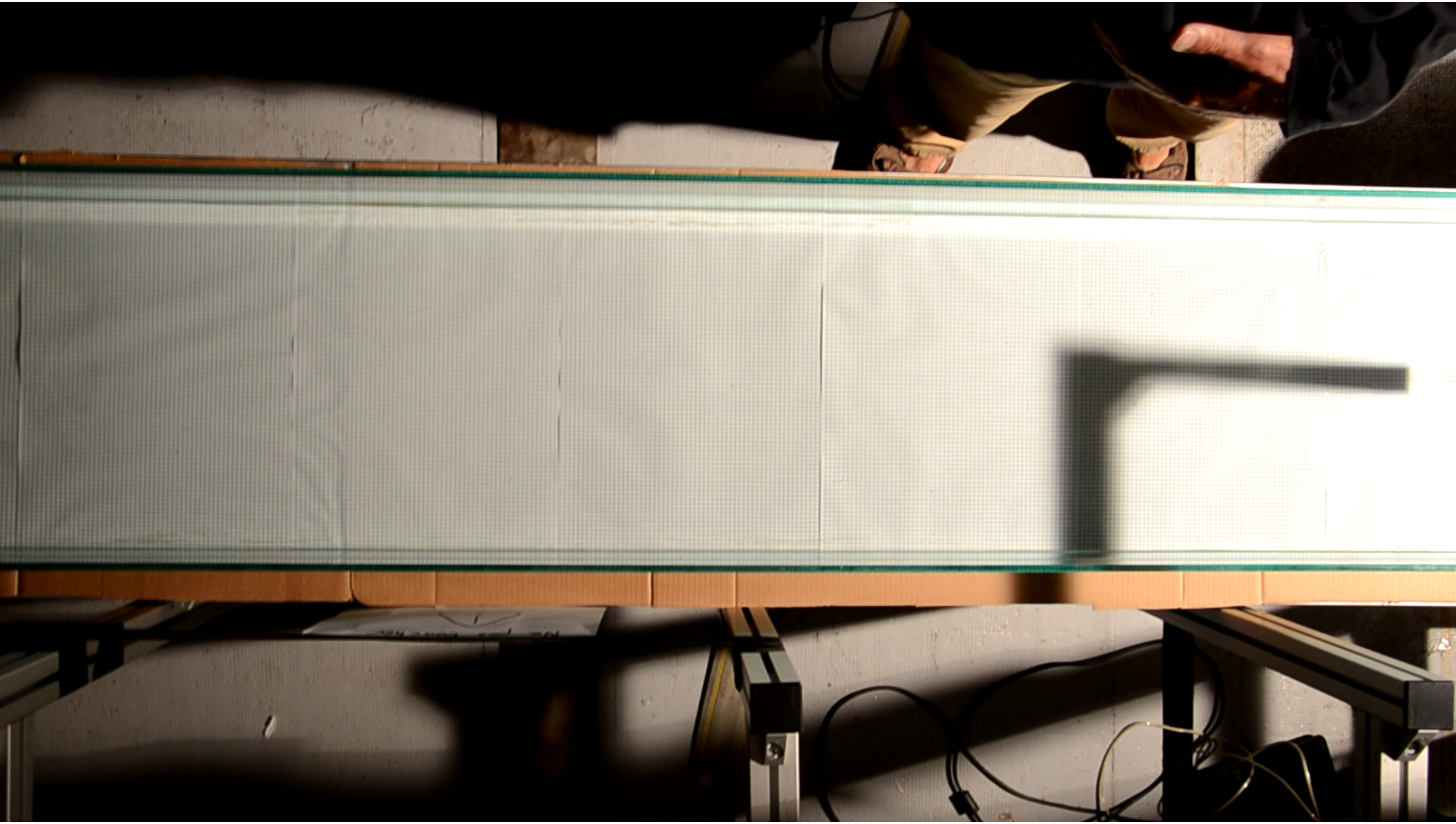


$\alpha \simeq 19°$

‘Cylindrical duck’ of diameter 3 cm, U = 2.6 m/s (Fr = 4.8)

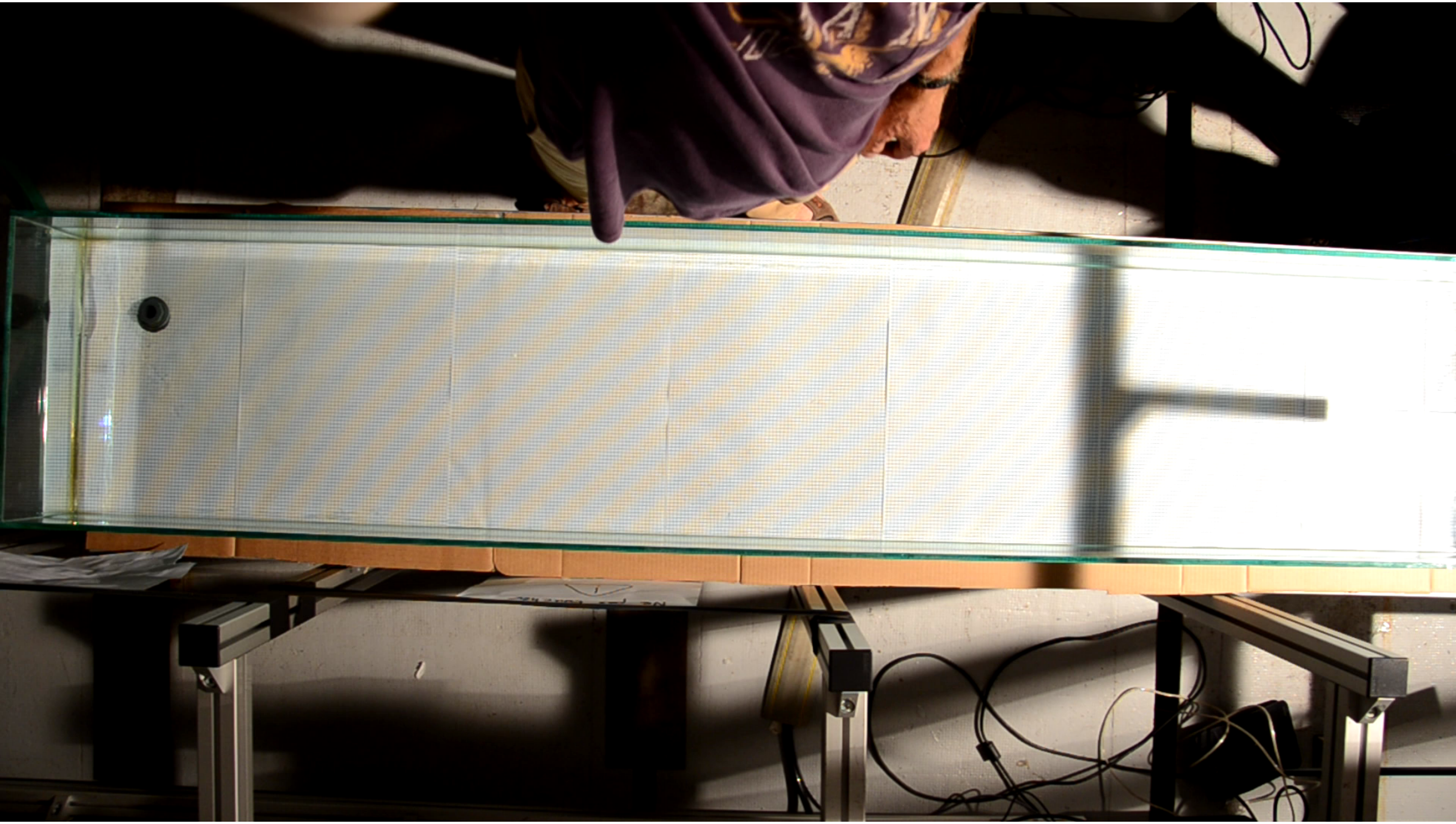
$\alpha \simeq 7.1°$

Cylinder of diameter 1.5 mm, U = 0.6 m/s (Fr = 4.9)



$\alpha \simeq 19°$

Cylinder of diameter 1.5 mm, U = 2 m/s (Fr = 16)

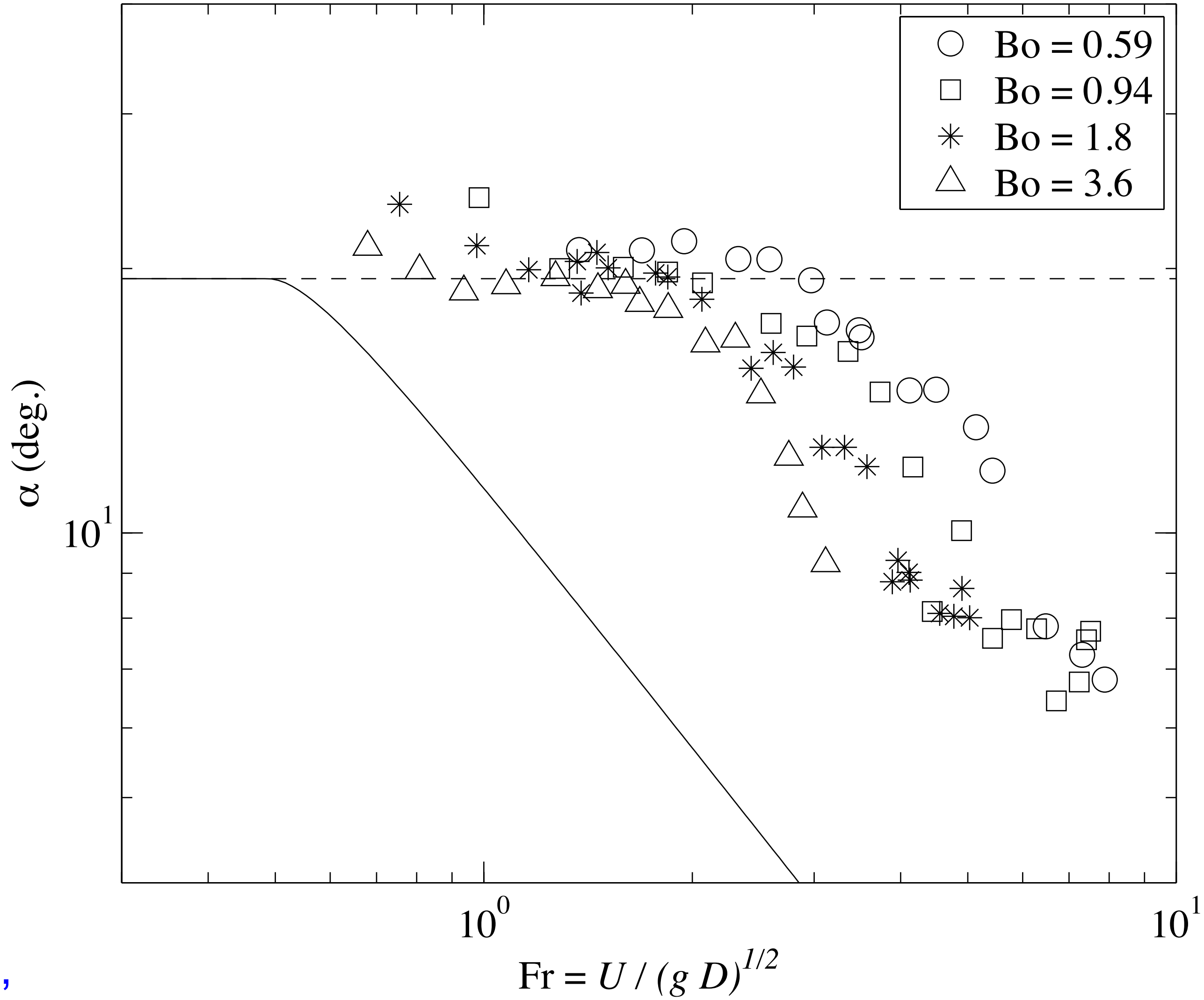


$$\alpha \approx 6^\circ$$

With capillary waves

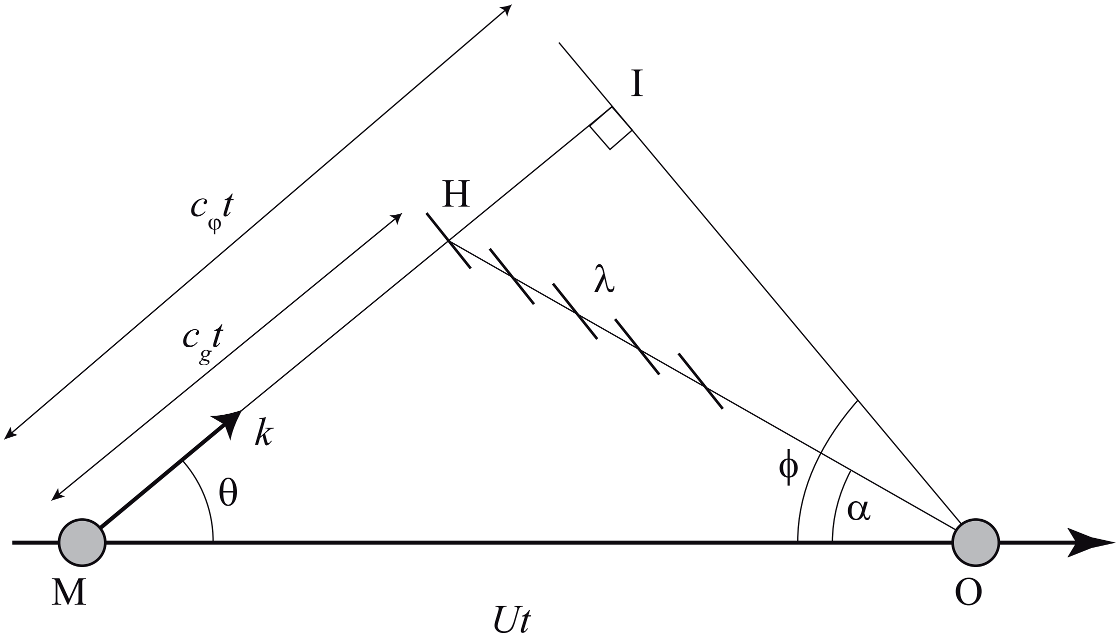
$$D \in [10, 16, 30, 62] \text{ mm}$$

$$Bo = \frac{D}{\lambda_c}$$



- 3 lengths D, λ_c and λ_g,
- 2 dimensionless number, Fr and Bo

With capillary waves



$$\omega(k) = \sqrt{gk + \frac{\gamma}{\rho} k^3}$$

$$U \cos\theta = c_\varphi = \frac{\omega}{k}$$

$$\tan\alpha(k) = \frac{c_g(k)\sqrt{U^2 - c_\varphi^2(k)}}{U^2 - c_g(k)\,c_\varphi(k)}$$

With capillary waves

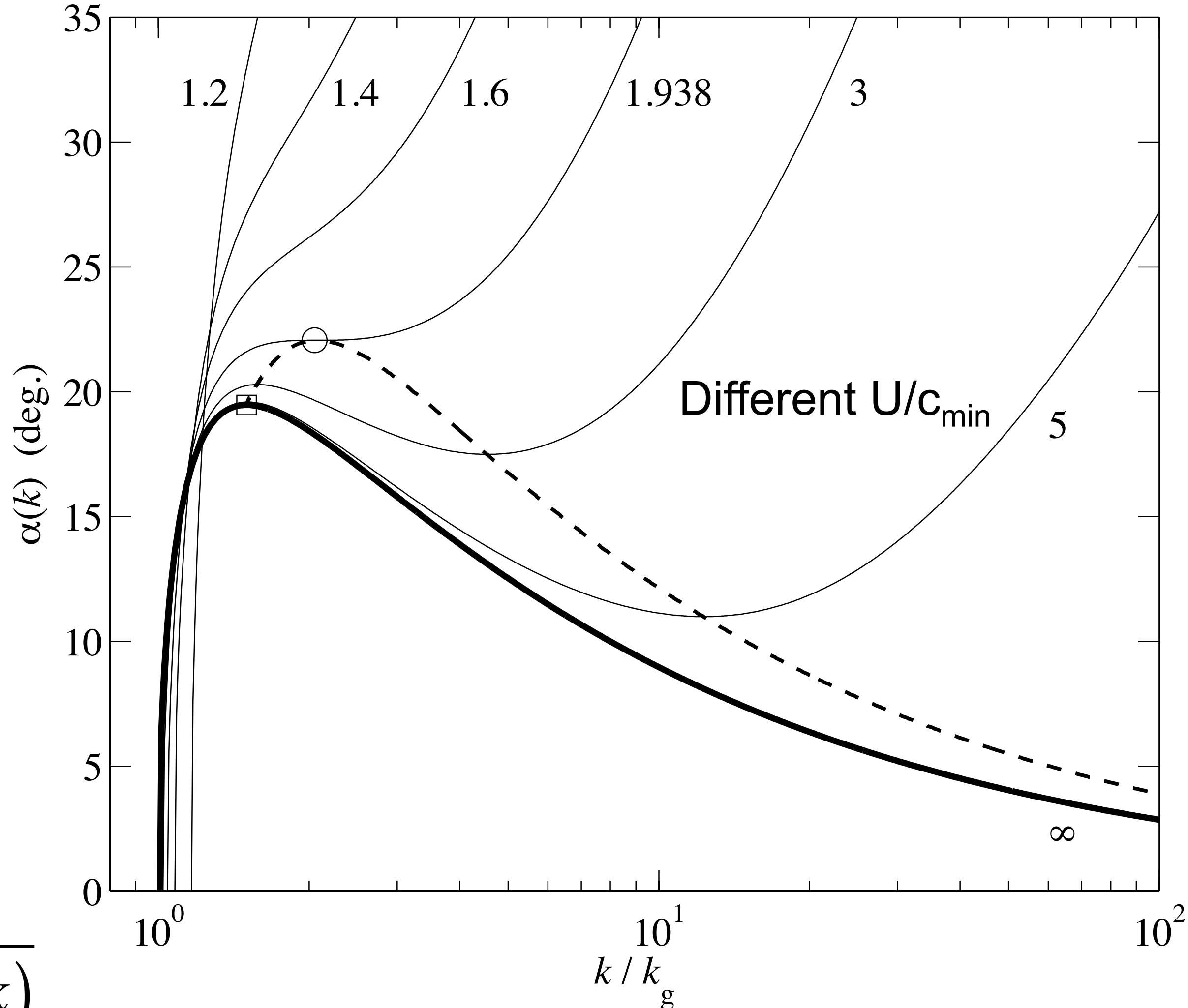
$$\frac{U}{c_{\min}} = Fr\sqrt{\pi Bo}$$

$$\omega(k) = \sqrt{gk + \frac{\gamma}{\rho}k^3}$$

$$U\sin\theta = c_\varphi = \frac{\omega}{k}$$

$$\tan\alpha(k) = \frac{c_g(k)\sqrt{U^2 - c_\varphi^2(k)}}{U^2 - c_g(k)c_\varphi(k)}$$

avec $k_g = \frac{g}{U^2}$



But damping in exp(-2νk²t)

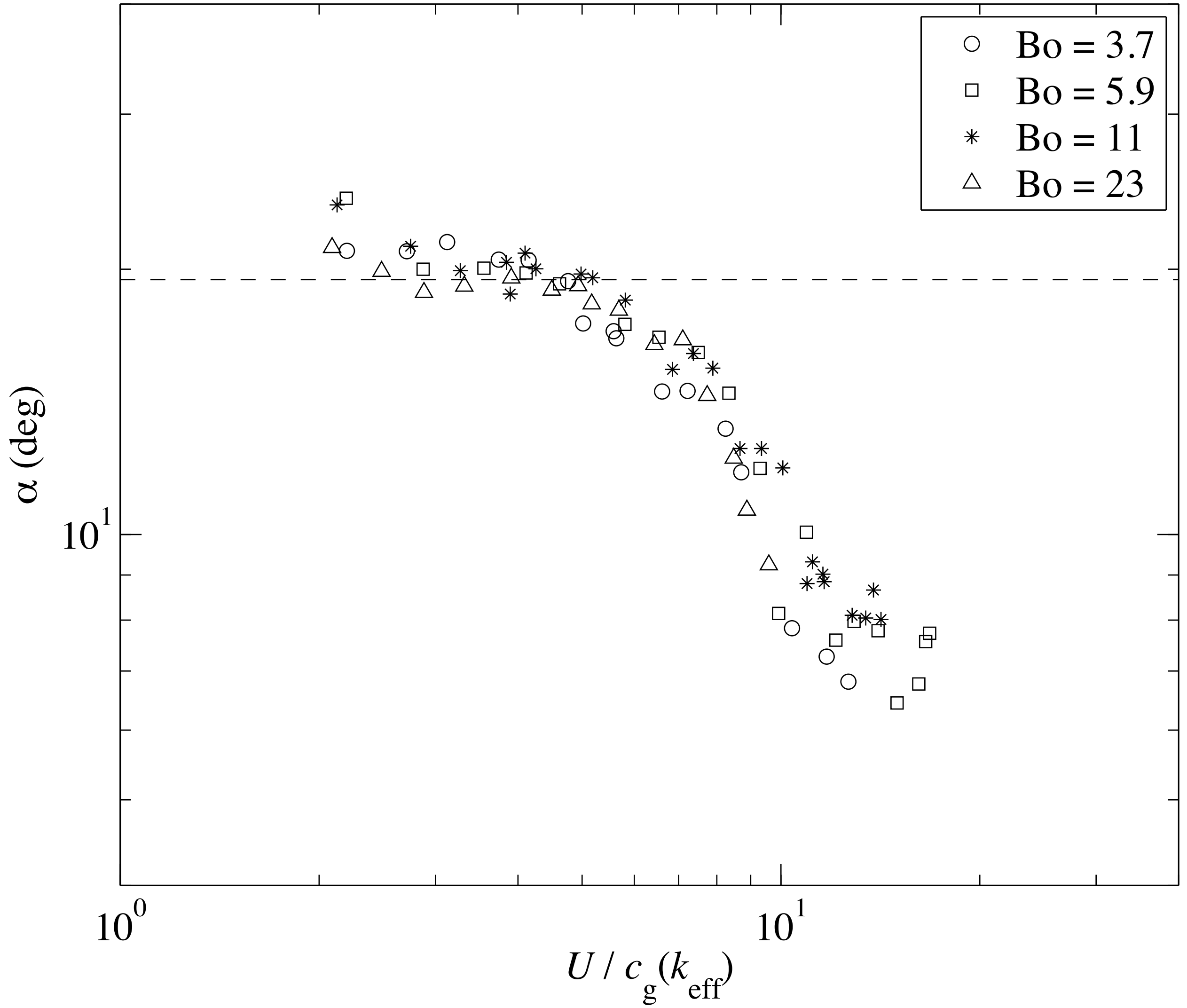
With capillary waves

$$D \in [10, 16, 30, 62] \text{ mm}$$

$$Bo = \frac{D}{\lambda_c}$$

Generalised Froude number:

$$Fr^* = \frac{U}{c_g(D)}$$



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Wave-making resistance

- Known since Froude

- Existence of a wave drag (dominant at large velocity)

$$R_w = \frac{1}{2}\rho U^2 B^2 C_w$$

Wave-making resistance

Havelock method (1918): Imposed pressure perturbation P(x,y)

$$R_W = \iint P(x,y) \frac{\partial \zeta}{\partial x} dx dy.$$

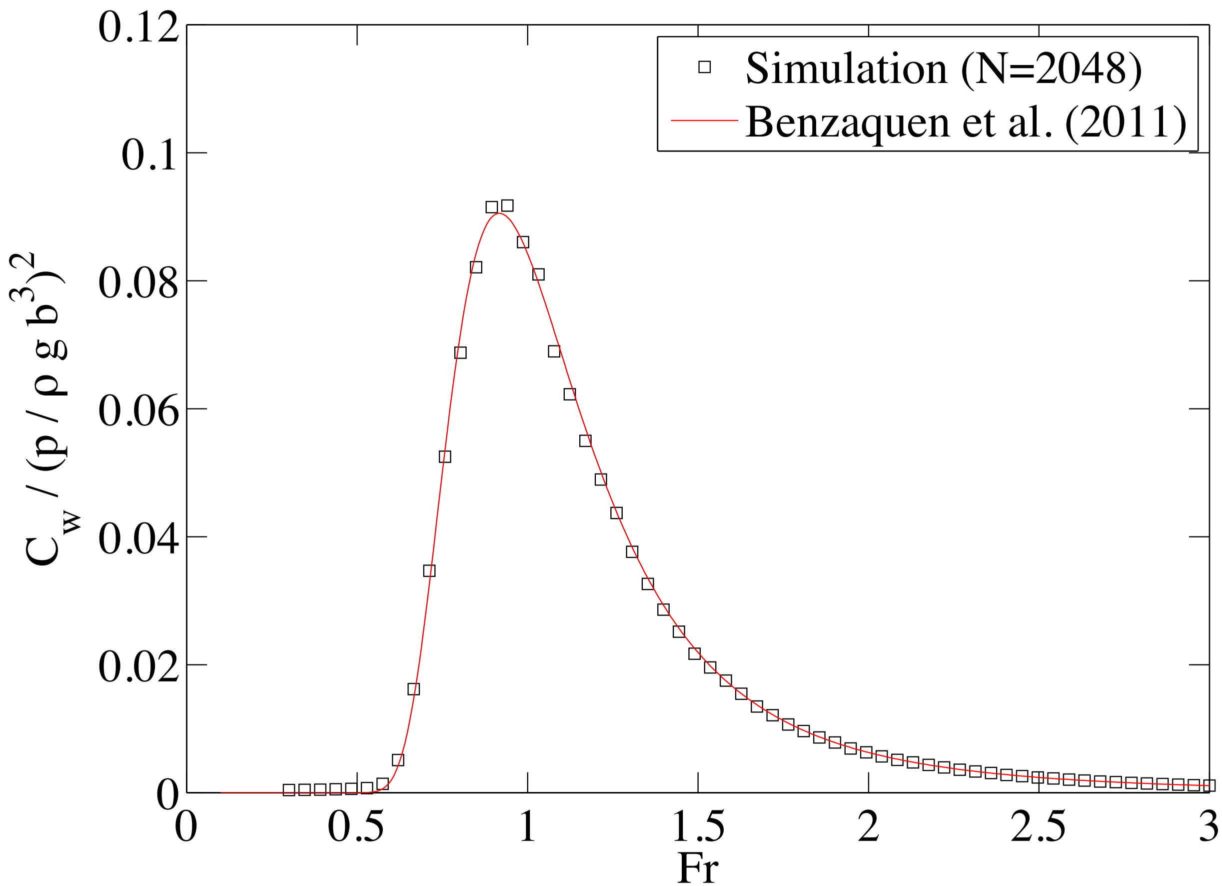
Where

$$\zeta(x,y) = \iint \frac{dk_x}{2\pi} \frac{dk_y}{2\pi} \frac{\hat{P}(k_x,k_y)}{\left[\gamma(k^2+k_c^2) - \frac{\rho}{k}(k_x U - i\epsilon)^2\right]} \exp i[k_x x + k_y y]$$

In the limit $\varepsilon \longrightarrow 0$

Wave-making resistance

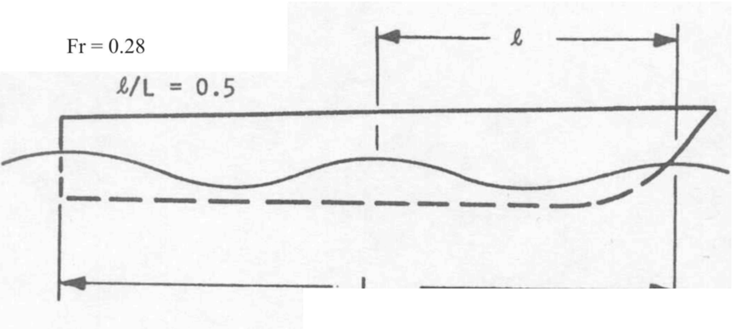
A decrease of the wave drag is found.

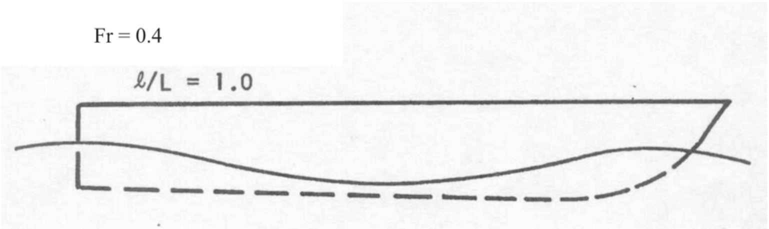


Analytical result of Benzaquen, Chevy, Raphaël for an imposed gaussian pressure field.

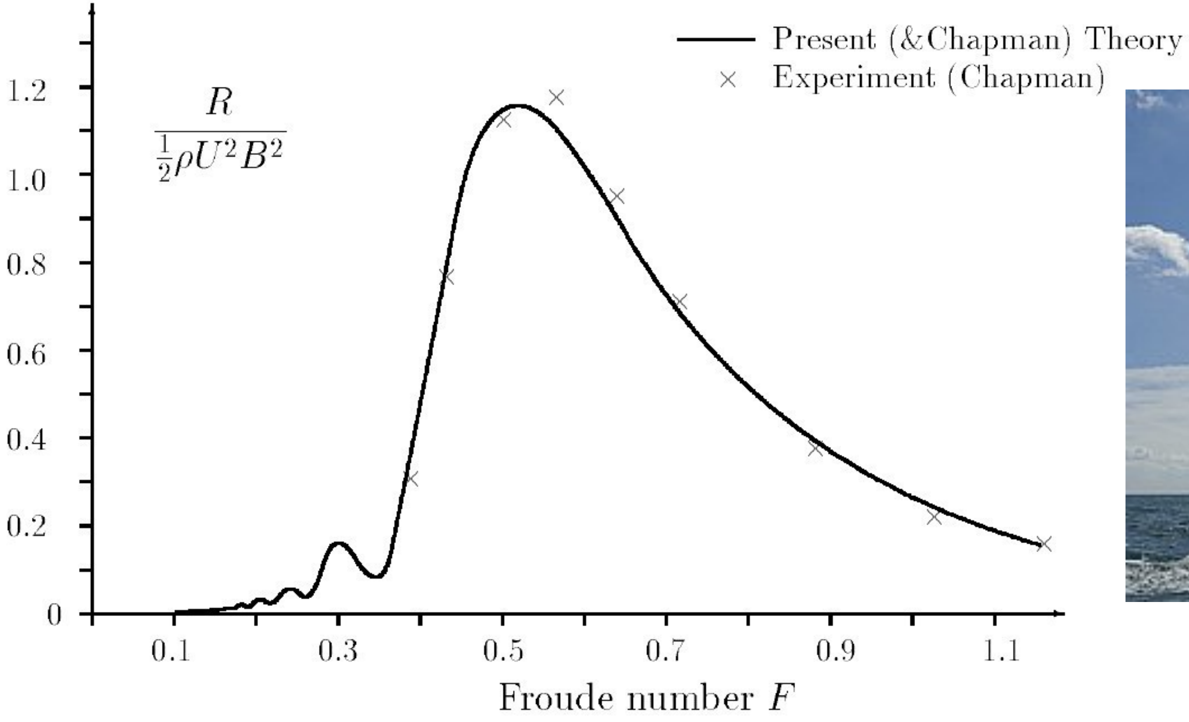
$$C_W = \left(\frac{D}{L^3}\right)^2 \frac{1}{\mathrm{Fr}^8} \int_0^{\pi/2} \frac{d\theta}{\cos^5\theta \exp\left[\left(\sqrt{2\pi}\mathrm{Fr}\cos\theta\right)^{-4}\right]}$$

Wave-making resistance





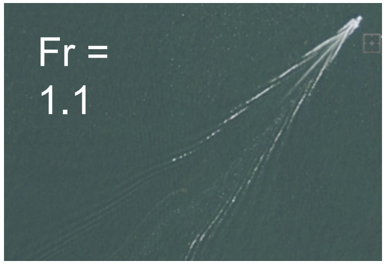
Wave resistance $R_w = f(Fr) \sim Fr^{6 \text{ or } 8}$



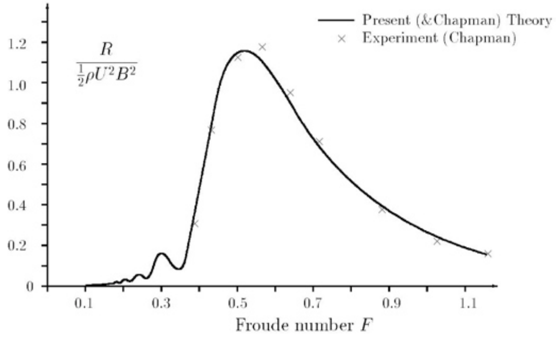
Planing hull

Planing Mini-Transat boat (Fr > 1)

Conclusions



- Narrow wave wakes exist, and are not explained by the classical Kelvin argument

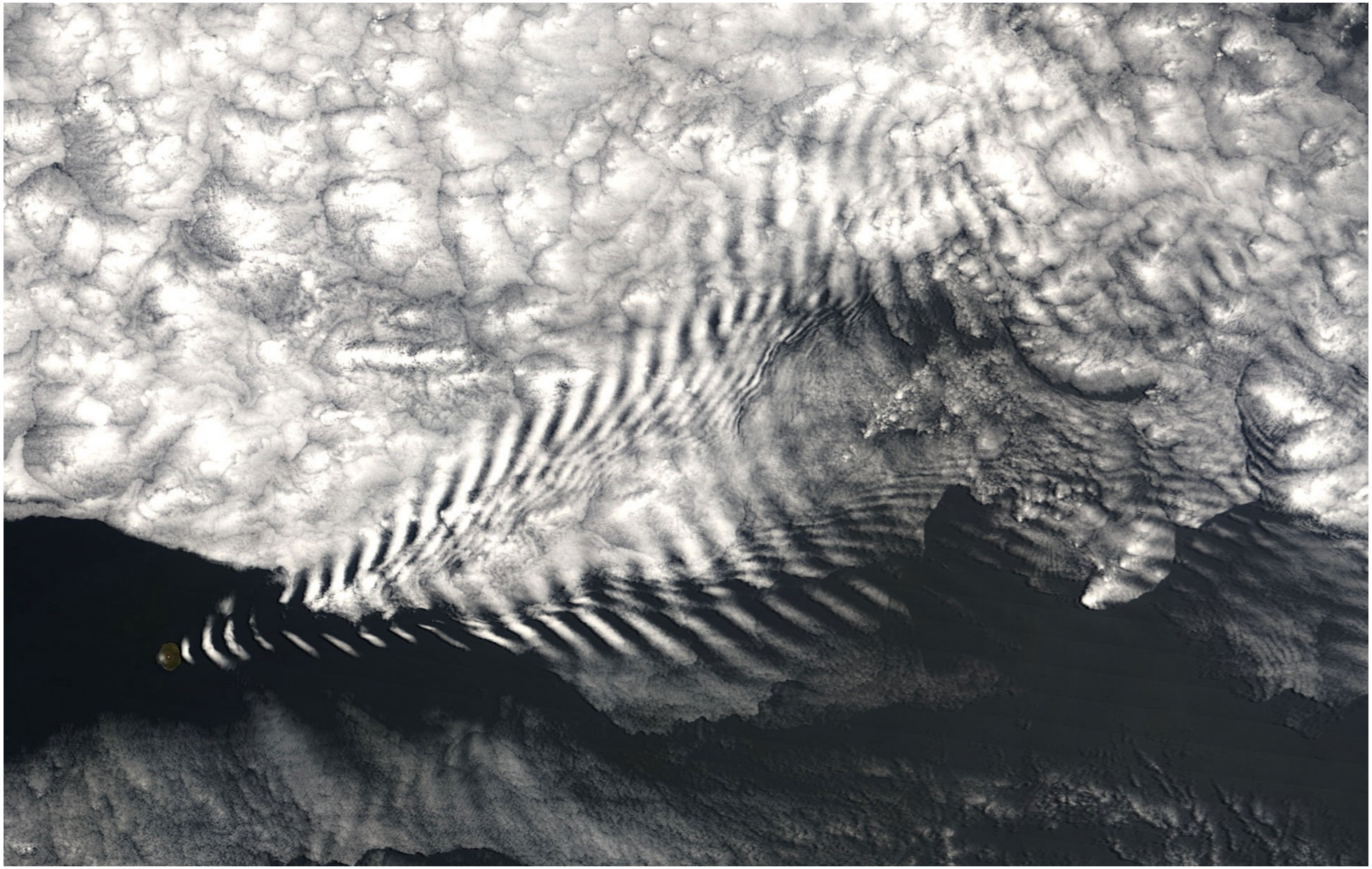


- Capillary regime is under progress.

- Fr > 1 => 1) Narrow Kelvin wake
2) Planing hull
3) a decrease of the wave drag

Is there a link or just a coincidence?

Thank you!



NASA satellite image (MODIS imager on board the Terra satellite) of a wave cloud forming off of Amsterdam Island in the far southern Indian Ocean. Image taken on December 19, 2005.

References

1. Thomson W. (Lord Kelvin), 1887, On ship waves, Trans. Inst. Mech. Eng., 409–433.

2. J. Lighthill. Waves in fluids. Cambridge University Press, Cambridge, 1978.

3. F.S. Crawford, Elementary derivation of the wake pattern of a boat, Am. J. Phys. 52, 782-785, 1984.

4. *M. Rabaud and F. Moisy,* Ship Wakes: Kelvin or Mach Angle?*, Phys. Rev. Letters 110, 214503 (mai 2013)*.