OPTICAL MEASUREMENT OF SHIP WAVES BY DIGITAL IMAGE CORRELATION

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SUMMARY

A large part of the drag experienced by fast sailing boats originates from the energy radiated at the sea surface by waves. Optimising hull shape in order to minimise wave resistance requires thus a good knowledge of the wave structure created by the ship as well as its evolution with boat heel and boat speed. This knowledge usually originates from numerical simulations or towing experiments.

We present here an optical method allowing the determination of the instantaneous 3D water surface shape with a good vertical resolution on large horizontal scales. This method may be useful in towing tank tests and should allow detailed comparisons with computed surface deformation and thus help to validate Velocity Prediction Programs.

NOMENCLATURE

Fr	Froude Number
U	Boat speed
g	Acceleration of the gravity
L_W	Length of the waterline
$\eta(x,y)$	Water elevation
$h(x,y)$	Local water eight
h_0	Mean water level with respect to the dot pattern
H	Distance from the camera to the mean water surface
U_c	Critical hull speed
$\delta\mathbf{r}$	Apparent displacement vector
PIV	Particle Image Velocimetry
A	Typical wave amplitude
λ	Typical wave length
n	optical index of air
n’	optical index of water
L	Length of the imaged area

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1. INTRODUCTION

A challenging domain in naval research is to design fast sailing boats minimizing the hull drag and more precisely the wave drag. Since the pioneering work of William Froude [1] and Havelock [2], it is known that the drag experienced by a boat increases rapidly when its velocity increases and that most of this increase originates from the energy dissipated in wave formation. Large amplitude waves generated by the motion of the hull are known to be localised in a wedge of half angle 19.5°, the so called Kelvin wedge [3], but the exact wave pattern is a function of the hull shape and orientation, and of the dimensionless Froude number, Fr = U/U_C, ratio of the boat speed to the critical hull speed:

$$U_c = \sqrt{\frac{gL_W}{\pi}}\,.$$

This critical velocity corresponds to the ship velocity for which the gravity waves moving with the ship have a wavelength equal to twice L_W the waterline of the boat. When the boat sails faster than this critical hull speed, Fr > 1, the wave resistance decreases, mainly because an increase of the sustentation forces changes the wetted volume of the hull. Most modern fast sailing boats attempt to navigate above U_c and a good prediction of their performance demands an accurate simulation of the generated wave pattern. These simulations are nowadays more and more accurate [3-6] but comparisons with tank tests are still necessary in particular for unstationary conditions when wind waves or swell are present.

Usual measurements of wave pattern are based on local measurement by resistive, ultrasonic or capacitive probes. These probes or array of probes measure the water level when the boat passes by. From the time evolution $h(t)$ the profile $h(x)$ is reconstructed assuming a constant velocity of the boat. The wave resistance can then be deduced by the longitudinal cut method of Sharma [7].

Direct measurement of one height profile at different instants was developed recently, a camera imaging the intersection of a laser sheet with the water interface [8]. However this method requires powerful lasers.

To our knowledge, no method until now was able to measure directly the full 3D water shape. In the present paper we thus address the possibility of measuring instantaneously the surface deformation $h(x,y)$ with a good horizontal and vertical resolutions using light refraction through the deformed surface. This technique was shown to be simple and very efficient at small scale [9] and a summary of these results is presented in the next section. We will discuss in section 3 the possibility to adapt this technique to measurement in towing tanks.

2. PRINCIPE OF THE METHOD

Our method is derived from PIV (Particle Image Velocimetry) technique, now currently used in fluid mechanics to determine fluid velocity in one plane.

2.1 EXPERIMENTAL SET-UP

In the present method a camera images randomly located dots (speckle pattern) printed on a plane screen through the air/water interface (Figure 1). A first reference image is taken when the interface is flat, and images taken when the interface is deformed are compared to the reference one.

The dots when seen through the deformed interface appear displaced on a small distance $\delta\mathbf{r}(x,y)$. To first order, if the camera is far enough and the slope of the interface is not too large, this displacement is proportional to the local slope **grad**(h) of the interface, to the air and water indexes *n* and *n'*, and to the water depth h_0:

$$\delta\mathbf{r} \approx -\frac{n'-n}{n'}\,h_0\,\mathbf{grad}(h)\cdot \qquad (1)$$

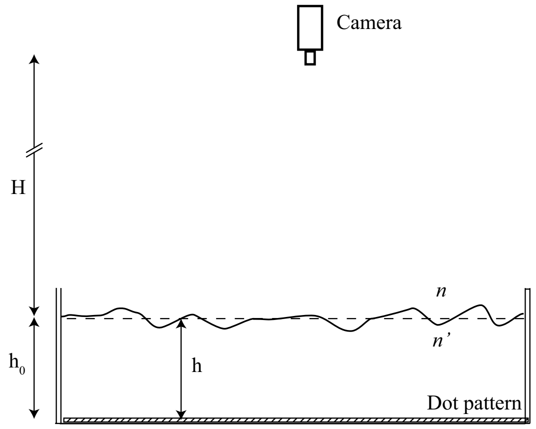

Figure 1: Experimental set-up showing the camera above the interface and the dotted screen located at the bottom of the tank.

2.2 DETERMINATION OF THE SURFACE SHAPE

The displacement field $\delta\mathbf{r}$ is obtained by processing the reference image and the deformed image using a standard PIV algorithm[2]. As the designed pattern is adjusted to reach the optimal dot concentration (about 50%) and the optimal dot size (few pixels), a subpixel resolution can be easily achieved.

Equation (1) can then be integrated numerically in order to obtain the surface height $h(x,y)$. Details of the integration method are given in Ref. 9.

Figure 2 gives an example of surface obtained in a test case with small amplitude plane waves. The visualised

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[2] *e.g.* LaVision GmbH, Anna-Vandenhoeck-Ring 19, 37081, Goettingen, Germany, complemented with the PIVMat toolbox for Matlab, http://www.fast.u-psud.fr/pivmat.

domain is of the order of 15 cm x 15 cm, with a horizontal resolution of 0.5 mm and a vertical resolution of the order of few micrometers (1% of the wave amplitude). Small transverse oscillations are clearly visible on the main waves.

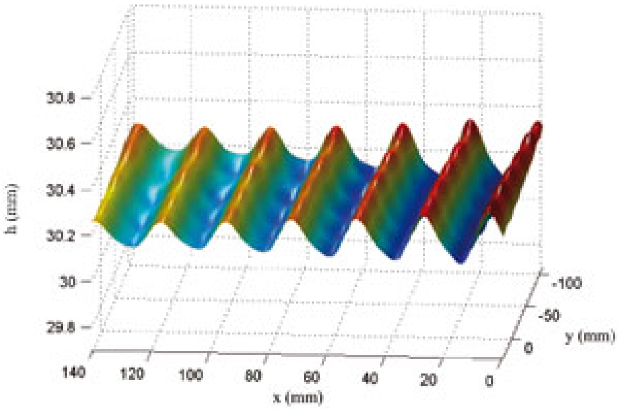

Figure 2: Typical surface shape measured with plane waves in a test experiment.

3. ESTIMATED ACHIEVABLE RESOLUTION AND LIMITATIONS FOR TOWING TANK MEASUREMENT

In a real towing tank, dimensions are very different from the ones of our test case. We selected typical values for a towing tank : $h_0 = 2$ m for the mean water depth, and an imaged surface of 4 m x 4 m.
With a 2048 x 2048 pixel camera imaging this surface, the pixel size would be of 2 mm. The typical size of the random dots on the bottom should then be of the order of 1 cm. Using PIV windows of 8 x 8 pixels, the x and y resolution will be of 16 mm, corresponding to 256 x 256 measurement points. As sub-pixel resolution is achievable, displacement of 0.2 mm should in principle be measurable.

In order to be far enough the camera must be located to an altitude H > 10 m for such large image. This is certainly a strong limitation if the ceiling of the towing tank is not high enough, however a 45° mirror could be add in order to use an horizontal optical axes.

With those order of magnitude, equation 1 predicts a slope resolution of 0.02°, and thus a vertical resolution of 6 µm. This excellent vertical resolution is possible because of the very good sensitivity of the PIV method and also because of the very large value of h_0. However this large value of the distance from the surface to the dot pattern has an important drawback: It will limit the maximum measurable curvature. Indeed the crest of the wave acts as a converging lens, and for monochromatic waves of amplitude A and wavelength λ, as the deformed surface writes $\eta = A\cos(2\pi x/\lambda - \omega t)$, the focal distance is $\lambda^2/(\pi^2 A)$. In order to avoid caustics or too large stretching of the dots this focal distance must remain much larger than the water depth h_0. If it is not the case, the displacement field is no more invertible [9]. This limits the measurable curvature of the interface:

$$A < \frac{\lambda^2}{10 h_0}. \qquad (2)$$

For example with $\lambda = 1$ m and $h_0 = 2$ m, Eq. (2) limits the maximum measurable wave amplitude to A < 5 cm.

This limitation on the wave amplitude can be overcome however by decreasing h_0 using an immersed dot screen located above the tank bottom. If this is not possible because of the draft of the hull or because it will alter the wave dispersion (shallow water approximation) the camera can also be put under water and the screen in air above the water surface. This method should then allow observing larger amplitude waves.

4. CONCLUSIONS

A technique to measure the full deformation of sea surface behind a sailing boat was described. The technique was already tested at small laboratory scale and show very good resolution. We believe that it can easily be adapted to larger scales without many difficulties, as it does not ask for peculiar material or critical optical skills.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

1. W. THOMSON (Lord KELVIN), ‘Popular Lectures and Addresses’, Vol. 3, 481-8, London: Macmillan. 1891
2. T. H. HAVELOCK, ‘The Propagation of Groups of Waves in Dispersive Media, with Application to Waves on Water Produced by a Travelling Disturbance’, *Proc. Roy. Soc. A, **81**, 398-430*, 1908
3. W. T. TSAI and D. K. YUE, ‘Computation of Nonlinear Free Surface Flows’, *Ann. Rev. Fluid Mech. **28**, 249–278*, 1996
4. E. PARAU, J-M. VANDEN-BROECK, ‘Nonlinear two- and three-dimensional free surface flows due to moving disturbances’, *European Journal of Mechanics B/Fluids **21**, 643–656*, 2002
5. E. O. TUCK, D. C. SCULLEN, and L. LAZAUSKAS, ‘Wave Patterns and Minimum Wave Resistance for High-Speed Vessels’, *24th Symposium on Naval Hydrodynamics, Fukuoka, Japan*, 8-13 July 2002
6. J. GARCIA and E. ONATE, ‘An Unstructured Finite Element Solver for Ship Hydrodynamics Problems’, *Transactions of the ASME. **70**, 18-26*, 2003

7. F. LALLI, F. DI FELICE, P. G. EXPOSITO, A. MORICONIA and R. PISCOPIA, 'Longitudinal cut method revisited: A survey on main error sources', *J. Ship Res.*, ***44***, *120-139*, 2000
8. NISHIO SHIGERU, NAKAO SHIGEKI and OKUNO TAKETOSHI, 'Image Measurement of the Wave Height Distributions around a Ship Hull in Regular Wave', *J. Soc. Naval Architects of Japan*, ***184***, *93-100*, 1998
9. F. MOISY, M. RABAUD, K. SALSAC, 'Measurement by Digital Image Correlation of the topography of a liquid interface', *submitted to Exp. in Fluids, download address* http://www.fast.u-psud.fr/~moisy/papers/moisy_eif07-preprint, 2008

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