Stabilizing viscosity contrast effect on miscible displacement in heterogeneous porous media, using lattice Bhatnagar–Gross–Krook simulations

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We analyze the displacement of a viscous fluid by a miscible more viscous one in heterogeneous porous media. We performed lattice Bhatnagar–Gross–Krook simulations, which were previously successfully applied to the study of the dispersion of a passive tracer in a stochastic heterogeneous porous medium. In the present situation, the flow is stable (no viscous fingering) and leads to an overall Gaussian dispersion, the coefficient of which decreases as the viscosity ratio increases. The results are in reasonable agreement with the stochastic approach of Welty and Gelhar. © 2004 American Institute of Physics. [DOI: 10.1063/1.1810474]

I. INTRODUCTION

One key issue in hydrology, contaminant remediation, and petroleum engineering is the understanding of the coupling between the porous medium heterogeneity and the fluid displacement properties given by buoyancy or viscosity effects. In this paper, we will focus on the viscous effect related to the displacement of a less viscous fluid in the porous medium by a more viscous fluid: This displacement is stable, as opposed to the well-known situation where a more viscous fluid is displaced by a less viscous one. If the latter case of viscous fingering has been extensively studied in homogeneous porous media (of permeability uniform in space), 1–3 it is not the case for realistic, i.e., heterogeneous porous media. A number of investigations 4–11 have addressed the issue of the coupling between a destabilizing viscosity contrast and the permeability distribution. Basically, in a heterogeneous porous medium, the fluid flows through the hydrodynamically easiest path that is through the larger permeability path, leading to an enhanced effect of heterogeneities, such as “resonance” between the intrinsic scale of the fingers (in homogeneous medium) and the correlation length of the heterogeneous porous medium. 3,6,9

Curiously enough, little attention has been paid to the case of stabilizing viscous effects in displacements in heterogeneous porous media. Let us mention an experiment in a layered porous medium, 6 where the resulting stratification of the displacement parallel to the layers was reduced and even suppressed for a large enough stabilizing viscosity ratio.

In the present paper, using our lattice BGK (Bhatnagar–Gross–Krook) simulation method, 12 well suited for tracer macrodispersion in heterogeneous porous media, 13–15 we address the issue of miscible stable displacements in more realistic heterogeneous porous media, namely, those with given log-normal permeability distributions and correlation lengths. We find that the mixing front between the fluids exhibits Gaussian dispersion, the dispersion coefficient of which decreases when the viscosity ratio increases. The results are shown to compare reasonably well with the extrapolation of the predictions by Welty and Gelhar 16 to our case.

II. NUMERICAL SIMULATIONS

As detailed in Ref. 12, the permeability field $K(\vec{r})$ of the porous medium was chosen to obey a log-normal distribution

$$\ln[K(\vec{r})] = \vec{f} + f'(\vec{r}),$$

where $\vec{f}$ is the mean of the distribution and where the perturbation field $f'(\vec{r})$ has a zero mean value and a variance $\sigma_f^2$. The isotropic spatial correlation of the permeability field is given by the exponential decay of the covariance function of $f'$.

$$R_{ff}(\vec{r}) = E[f'(\vec{r})f'(\vec{r} + \vec{\xi})] = \sigma_f^2 \exp \left( -\frac{\vec{r}}{\lambda} \right)$$

with $\lambda$ the correlation length. Such a porous medium was generated by convolving a random white noise field with an ad hoc correlation function, namely,

$$h(x,y) = \exp[(x^2/\delta^2 + y^2/\delta^2)^{1/3}],$$

where the value of the parameter $\delta$ sets that of the correlation length $\lambda$. Note that the characteristic value of the resulting permeability field is $K_l = \exp(\vec{f})$.

We then study the displacement in the porous medium of a fluid of viscosity $\mu_1$ by another fluid of viscosity $\mu_2$, miscible to the first one, and flowing at the constant mean rate $q_0$ in the $x$ direction. To achieve this, we perform the simulation of the following equations for the velocity field $\vec{q}$ and the concentration $C$ of the injected fluid:

$$\vec{\nabla} \cdot \vec{q} = 0,$$

$$\vec{\nabla} P = -\frac{\mu}{K(\vec{r})} \vec{q} + \mu \Delta \vec{q},$$
\[
\frac{\partial C}{\partial t} + \vec{q} \cdot \vec{\nabla} C = D_m \Delta C.
\] (6)

We note that the flow equation [Eq. (5)] involves a Brinkman-like term, \(\mu \Delta \vec{q}\), the influence of which on tracer macrodispersion was analyzed in a previous paper.12 Following Refs. 16 and 17, the fluid viscosity \(\mu\) is assumed to have an exponential concentration dependence, namely,

\[
\mu(C) = \mu_1 \exp(\beta C),
\] (7)

where

\[
\beta = \ln \left( \frac{\mu_2}{\mu_1} \right).
\] (8)

The case \(\beta>0\) corresponds to a stable displacement, whereas \(\beta<0\) corresponds to an unstable one (viscous fingering). Note also that the concentration \(C\) undergoes an isotropic mesoscopic diffusion of coefficient \(D_m\) [Eq. (6)], leading to the same value of the mesoscopic longitudinal and transverse dispersivities: \(\alpha_L = \alpha_T = D_m/q_0\).

The simulations presented here were performed on typical mesh sizes \(512 \times 512\) and during \(150,000\) time steps, using a 1.7 GHz Pentium IV. They were characterized by a Brinkman parameter \(K_b/\lambda^2 \approx 1\), which minimizes the effect of the Brinkman term, and a variance \(\sigma^2\) of the permeability distribution ranging in \([0, 1]\). Typical mean flow rate \(q_0\), viscosity ratio \(\mu_2/\mu_1\) (and thus \(\beta\)), and diffusion coefficient \(D_m\) ranged in \([0.001, 0.01], [1, 2.5] (\beta \in [0, 0.9])\) and \([5 \times 10^{-5}, 10^{-3}]\), respectively. In such conditions, the CPU time was about 10 h.

Figure 1 (top) shows typical invasion patterns, on the same permeability distribution, for tracer dispersion (\(\mu_2/\mu_1 =1\), \(\beta=0\), left) and for a stable displacement (\(\mu_2/\mu_1 =1.22\), \(\beta=0.2\), right). Even for this viscosity ratio close to 1, the sharpening of the mixing front is clearly observed, compared to the tracer case. An estimate of the front width is obtained by computing the derivative of the mean concentration profile \(\tilde{C}(x)\) (dots in Fig. 1, bottom)). Then the so-obtained mean concentration gradients are fitted tentatively to Gaussian profiles (lines), as functions of the distance \(x\) to the injection plane.

\[
\tilde{C}(x) = 0.5 \left( 1 - \frac{x}{a}\right) \exp \left( -\frac{x^2}{2a^2} \right).
\]
This diffusive behavior was observed whatever the values of \( q_0, \beta, D_m, \) and \( \sigma_f \) used (in the ranges given above): This supports the contention that the mixing regime is diffusive in our range of parameters.

III. THEORY

Before proceeding to the data analysis, let us summarize the analytical derivation of the macroscopic dispersion coefficient in a viscously stabilizing two-dimensional displacement in a heterogeneous porous medium. For this purpose, we follow the same approach as in Refs. 16 and 12 and use the permeability field [Eqs. (1) and (2)] and the flow equations [Eqs. (4), (5), (7), and (8)] detailed above. The diffusion equation of the concentration \( C \) is written in the form

\[
\frac{1}{2} \frac{d \sigma^2}{dt} = D_{\text{eff}}.
\]

This diffusive behavior was observed whatever the values of \( q_0, \beta, D_m, \) and \( \sigma_f \) used (in the ranges given above): This supports the contention that the mixing regime is diffusive in our range of parameters.

IV. RESULTS AND DISCUSSION

The viscosity effects can be quantified by measuring the effective diffusion coefficient \( D_{\text{eff}} \) in terms of \( b \). As mentioned above, the value of \( b \), which is proportional to the concentration gradient, depends on the location where the latter quantity is measured. Here, we arbitrarily chose the largest concentration gradient in the middle of the front (see Fig. 1), and hence the largest parameter \( b \), namely,

\[
b_M = -\frac{b \lambda^2}{\alpha_T} \left. \frac{\partial \bar{C}}{\partial x} \right|_{\text{Max}}.
\]

Note that in so doing, the theoretical dispersion coefficient [Eq. (10)] is underestimated.

Figure 3 shows the effective diffusion coefficient \( D_{\text{eff}} \) normalized by the value, \( q_0 \lambda \sigma_f^2 \), obtained in the tracer case.


\[
(\beta=0 \text{ and for } K_i/\lambda^2 \to 0), \text{ versus the parameter } b_M. \text{ Note that each symbol in Fig. 3 corresponds to the variation of one control parameter (} q_0, \beta, D_m, \text{ or } \sigma^2) \text{ at a time, and that each data point is obtained from one given realization of the porous medium, which induces some scatter in the results. However, the data collapse reasonably well on a single curve. This supports the idea that the front experiences a uniform effective mobility gradient. Figure 3 also displays the theoretical prediction of Eq. (10) (solid line), using the same value } b_M \text{ as in the simulations. Although the trend is the same as for the simulations, the model underestimates the diffusion coefficient, in accordance with the above remark. Therefore, we also plot in Fig. 3 the analytical curves obtained for smaller mean concentration gradients, corresponding to } b=b_M/2 \text{ and } b=b_M/3. \text{ A better agreement is observed when a mean concentration gradient, typically two times as small as the maximum gradient, is used in the prediction of Eq. (10). Note that as diffusion proceeds, the concentration gradient should decrease in time, leading to a decrease of } b. \text{ Consequently, the diffusion coefficient predicted by the model should increase in time, leading eventually to the dispersive nature of the tracer case, } \alpha(b=0), \text{ as in Ref. 12. However, the characteristic time for such a process is far beyond our simulation means. For instance, in the conditions of the simulation of Fig. 1, right, the variance } \sigma^2(t) \text{ should reach the value of } 10^6 \text{ (compared to a few hundreds at the end of the simulation) for the macrodispersivity of the tracer case to be obtained. We may note also that the increase of the diffusion coefficient in time is a direct consequence of the transverse diffusion, which tends to smooth out the viscosity contrast. This can be compared to the well-known Taylor diffusion in a capillary tube, where the transverse dispersion is responsible for the diffusive regime at long times and to miscible displacements between two plates, in which, for small Peclét numbers and a stable viscosity ratio, a diffusion regime was obtained with a diffusion coefficient equal to that of the Taylor regime (tracer case).}

V. CONCLUSION

We studied in this paper the macrodispersion in miscible and stable displacements of a less viscous fluid by a more viscous one in heterogeneous porous media, by means of a lattice BGK simulation method. This method was previously applied to the study of the effect of the Brinkman parameter, \( K_i/\lambda^2 \), on the macrodispersion of a passive tracer in a stochastic heterogeneous porous medium. The present work focused on the quantitative estimation of the stabilizing viscous effects on macrodispersion. We showed that, after some transient time, a diffusivelike mixing regime was reached, the effective diffusion coefficient of which depended on the so-called viscous parameter \( b = - (\beta \lambda^2 / \alpha_p) (d \bar{C} / dx) \), involving the porous medium correlation length \( \lambda \), the viscosity contrast \( \beta \), the transverse dispersion \( \alpha_p \), and the mean concentration gradient \( -(d \bar{C} / dx) \) of the injected fluid, as derived in Ref. 16. Although the assumption of a uniform mean concentration gradient of the model was not verified, it was shown that the behavior of the diffusion coefficient as a function of \( b \) in the numerical simulations agreed with the model prediction [Eq. (10)], provided that a value of about half the maximum of the observed gradient was taken to evaluate \( b \).

This work will be extended, in stochastic heterogeneous porous media, to the case of viscosity unstable miscible displacements, and to the interplay between the resulting viscous fingering and the effect of the underlying porous medium heterogeneities.

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