# Turbulent mixing of a passive scalar: Statistics of the cliffs

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The small scale persistence of anisotropy in turbulent mixing of a passive scalar is known to be related to the presence of cliffs [1], large-scale scalar jumps concentrated over small separations. However the link, if any, between cliffs and inertial range anomalous scaling remains far from clear. We report here a detailed study of cliff statistics, obtained from one-point temperature time series in a high-Reynolds number turbulence experiment in low temperature helium gas. We will focus on statistics of spatial organization of high temperature gradients, giving evidence of self-similar clustering for inertial range separations.

The flow takes place in a cylindrical vessel, 20 cm in diameter, and is generated between two coaxial disks, 13 cm apart [2], rotating in the same direction. The fluid is helium gas at a temperature of 8 K and at pressures ranging from 0.3 to 3 bar. Thermal fluctuations, of typically 50 mK, are induced by the means of a heated grid, and temperature measurements are performed at a position 30 mesh sizes downstream. The hand-made thermometers are 7 $\mu$m diameter carbon fiber, working in constant current mode, with a resolution of 100 $\mu$K.

For this first study of temperature measurements we restrict ourself to a range of microscale Reynolds numbers $R_\lambda$ from 100 to 300. The turbulence parameters are determined from velocity measurements at the same point and in the same flow configuration. The integral length scales of temperature ($\theta$) and velocity ($u$) fluctuations are respectively $\Lambda_\theta = 0.7$ cm and $\Lambda_u = 1.0$ cm, with no noticeable $R_\lambda$ dependence, and the Kolmogorov scale $\eta$ lies between 185 and 39 $\mu$m.

The thermal cliffs are defined from a simple threshold on the temperature derivative,

$$|\partial\theta/\partial x| > s\,\langle(\partial\theta/\partial x)^2\rangle^{1/2},$$

where $s$ is an arbitrary constant, in the range 3–20. Spatial derivatives are obtained from temporal ones using the Taylor hypothesis. Figure 1a shows a magnification of a cliff, concentrating a temperature jump of about 5 standard

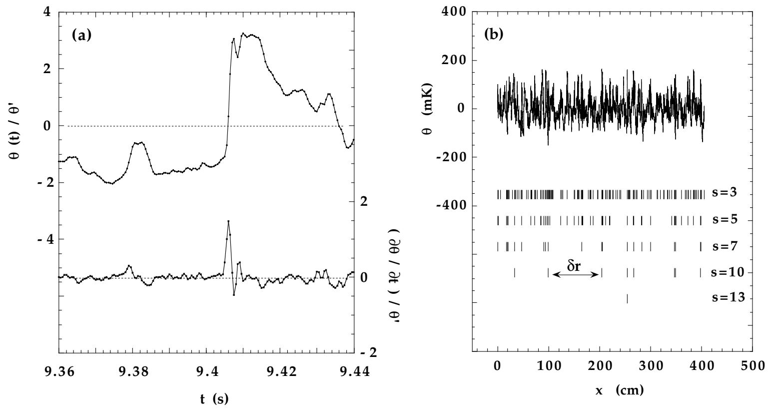


Figure 1: (a) Detail of a typical cliff on the temperature signal, and its corresponding time derivative. (b) Long recording of temperature fluctuations; the bars indicate the positions of the cliffs, for different thresholds, revealing the hierarchical organization of cliffs.

deviations. The cliff width, $\Delta$, is defined such that the temperature derivative takes values exceeding 90 % of its local maximum. Distributions of this width are shown in figure 2a, for three different thresholds, at a Reynolds number $R_\lambda = 280$. It is clear that no threshold dependence appears, giving confidence in our detection method. Figure 2b shows the mean cliff width divided by the Kolmogorov scale for different Reynolds number from 100 to 300. We can see a well defined plateau,

$$\langle \Delta \rangle = (13 \pm 3)\eta.$$

Although a $\lambda \sim R_\lambda^{-1}$ scaling cannot be ruled out because of the small range of Reynolds numbers spanned here, our data suggest that the $\eta \sim R_\lambda^{-3/2}$ scaling is more likely. This observation indicates that the highest scalar jumps, whose amplitude is a few temperature standard deviations, are concentrated over distances scaling as the Kolmogorov scale, the "smallest available lengthscale" of the flow.

In order to investigate the occurrence of cliffs in the temperature signal, we now focus on the statistics of intervals between cliffs. In figure 1b we show a long recording of temperature fluctuations together with the positions of the cliffs, indicated by vertical bars for different thresholds. From this plot, a hierarchical organization of cliffs appears: the strongest gradients (highest $s$) are surrounded by weaker ones (smaller $s$). This clustering trend appears more clearly if we plot the probability density function (pdf) of these interval lengths, as shown in figure 3a. These pdfs are very wide, so we have used width-varying bins to compute the histograms. For long separations, the pdf is well fitted by an exponential decay, a signature of uncorrelated events. The characteristic length

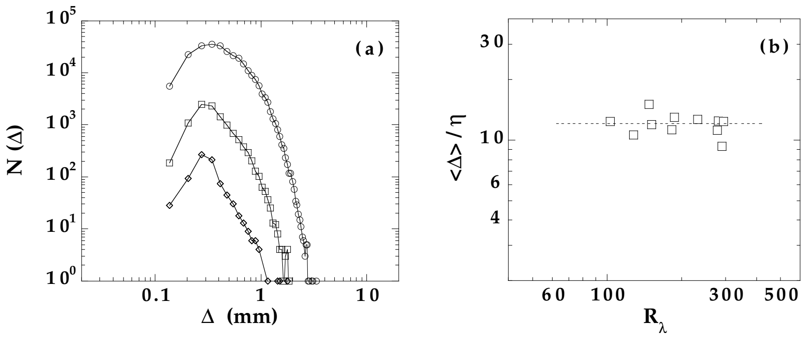


Figure 2: (a) Histograms of cliff width $\Delta$, defined with thresholds $s$=3, 7 and 13. (b) Mean cliff width, divided by the Kolmogorov scale $\eta$, as a function of $R_\lambda$.

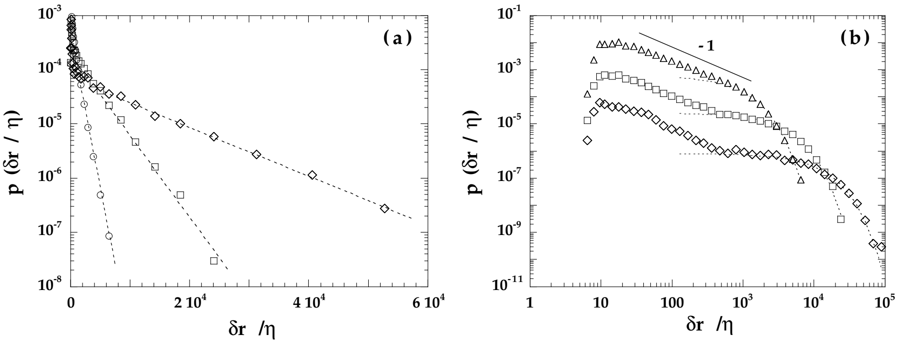


Figure 3: Distribution of interval between fronts for different thresholds $s$=3, 7 and 13, in log-linear (a) and log-log (b) coordinates. The dashed lines are exponential fit for $\delta r/\eta > 500$.

scale of this exponential behavior is related to the ratio of events selected by the threshold $s$.

For smaller separations, the pdf shows an algebraic decay,

$$p(\delta r) \sim \delta r^{-\mu},$$

clearly visible in figure 3b, with an exponent $\mu$ close to -1. The logarithmic derivative of this pdf gives a local exponent $\mu = 0.98 \pm 0.05$ for $30 < \delta r/\eta < 300$, with no variation with the threshold $s$. For higher thresholds, fewer events are selected and the exponential contribution dominates the algebraic one. The cross-over between these two regimes is $L_c \simeq (2.4 \pm 0.1)\Lambda_\theta$, again with no threshold dependence.

This algebraic distribution of waiting times between cliffs strongly suggests a self-similar clustering, in which the only characteristic size appears to be the upper bound of the inertial range. One consequence of such a law is that the mean interval between cliffs, $\langle \delta r \rangle$, is fixed by the large scale. Together with the mean cliff width $\langle \Delta \rangle \sim \eta$ mentioned above, we note that this result is in good agreement with the constant $\sim o(1)$ skewness of temperature derivative observed at high Reynolds numbers.

Algebraic distributions of waiting times between intense events have been observed for other turbulent quantities. Thresholds on pressure drops in the turbulent flow between counter-rotating disks [3] reveal an algebraic distribution for short waiting times, with an exponent $\mu \simeq 1.6$ up to separations close to the integral scale. Waiting times between successive intense velocity bursts in the near field of a turbulent jet [4] also show algebraic distributions. For thresholds performed on longitudinal velocity derivative [5], we observe algebraic distributions with an exponent $\mu$ increasing from 0.5 to 1 for $R_\lambda < 400$ (in agreement with earlier observations [6] at moderate Reynolds numbers), and saturating at the value $\mu \simeq 1$ for higher $R_\lambda$ (up to 2000).

Such distributions reveal the hierarchical organization of the small scale structures of turbulent flows, highlighting the intermittent behaviour of energy and scalar dissipation. An exciting issue is the universality of the $p(\delta r) \sim \delta r^{-1}$ law of waiting times, for both scalar and velocity derivatives, and its link with the inertial range anomalous scaling of scalar and velocity structure functions.

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