ANISOTROPIC ENERGY TRANSFERS IN DECAYING ROTATING TURBULENCE

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1 Introduction

In the presence of a background rotation, a situation which is relevant for most geophysical and astrophysical flows, the energy cascade from large to small scales is modified by the Coriolis force, resulting in a gradual columnar structuring of the turbulence along the rotation axis. In the limit of large rotation rates, turbulence tends to become two-dimensional —but still three-component (2D-3C)—, in agreement with the Taylor-Proudman theorem.

The anisotropic energy transfers responsible for this non-trivial flow organization have been characterized mainly in the spectral space [1, 2, 3]. On the other hand, direct evidence of the anisotropy of the energy transfers in the physical space is still lacking.

If homogeneity (but not necessarily isotropy) holds, the energy transfers in the physical space are governed by the Kármán-Howarth-Monin (KHM) equation [4, 5]

$$\frac{1}{2}\frac{\partial}{\partial t}R = \frac{1}{4}\nabla\cdot\mathbf{F} + \nu\nabla^2 R, \tag{1}$$

where $R(\mathbf{r},t) = \langle \mathbf{u}(\mathbf{x},t)\cdot\mathbf{u}(\mathbf{x}+\mathbf{r},t)\rangle$ is the two-point velocity correlation, $\mathbf{F}(\mathbf{r},t) = \langle \delta\mathbf{u}\,(\delta\mathbf{u})^2\rangle$ is the energy flux density, and $\delta\mathbf{u} = \mathbf{u}(\mathbf{x}+\mathbf{r},t) - \mathbf{u}(\mathbf{x},t)$ is the velocity vector increment over separation $\mathbf{r}$. Eq. (1) reduces to the Kolmogorov's 4/5th law in the inertial range if isotropy is assumed. Importantly, this equation is still valid for homogeneous anisotropic turbulence, and in particular for axisymmetric turbulence in a rotating frame [6].

2 Experiment

Experiments of decaying grid turbulence mounted on the "Gyroflow" rotating platform have been performed [7, 8]. Turbulence is generated by rapidly towing a square grid at a velocity $V_g = 1.0$ m s^{-1} from the bottom to the top of a tank filled with 240 liters of water. The grid consists in 8 mm thick bars with a mesh size $M = 40$ mm. Runs for three rotation rates, $\Omega = 4$, 8 and 16 rpm, as well as a reference run without rotation, have been carried out. The Reynolds number based on the grid mesh is $Re_g = V_g M/\nu = 40\,000$, and the Rossby number $Ro_g = V_g/2\Omega M$ ranges from 7.4 to 30, indicating that the flow in the wake of the grid is fully turbulent and weakly affected by rotation.

Velocity measurements are performed in the rotating frame using a corotating PIV system. Two velocity components (u_x, u_z) are measured, in a vertical 16×16 cm^2 field of view, where z is the rotation axis. From these 2D PIV fields, surrogates of the energy distribution and flux density are computed,

$$E(\mathbf{r}) = \langle \delta u_x^2 + \delta u_z^2\rangle_{x,z}, \quad \mathbf{F}(\mathbf{r}) = \langle \delta\mathbf{u}(\delta u_x^2 + \delta u_z^2)\rangle_{x,z}. \tag{2}$$

Statistics are averaged over 600 independent realizations of the turbulence decay, ensuring a convergence of order of 20% for $\mathbf{F}(\mathbf{r})$, and better than 1% for $E(\mathbf{r})$.

3 Results

Our main findings are summarized in figure 1. In the absence of rotation (Fig. 1a), the raw vector map of the energy flux density $\mathbf{F}(\mathbf{r})$ is found nearly radial, pointing towards the origin, giving direct evidence of the isotropic energy cascade in the physical space, from the large to the small scales. The map of the energy flux $\nabla\cdot\mathbf{F}$ is remarkably circular, showing a broad negative minimum in an annular region spanning over $r \simeq 5 - 20$ mm, providing an indication of the extent of the inertial range.

Surprisingly, the flux density $\mathbf{F}(\mathbf{r})$ in the rotating case (Fig. 1b) is also nearly radial, except at the smallest scales, for $r < 10$ mm, where a marked deflection towards the rotation axis is observed. Such horizontally tilted $\mathbf{F}$ is indeed consistent with an asymptotic 2D-3C flow, for which $\mathbf{F}$ must be a strictly horizontal vector, function of the horizontal component of the separation only. The inertial range, where the energy flux $\nabla\cdot\mathbf{F}$ is

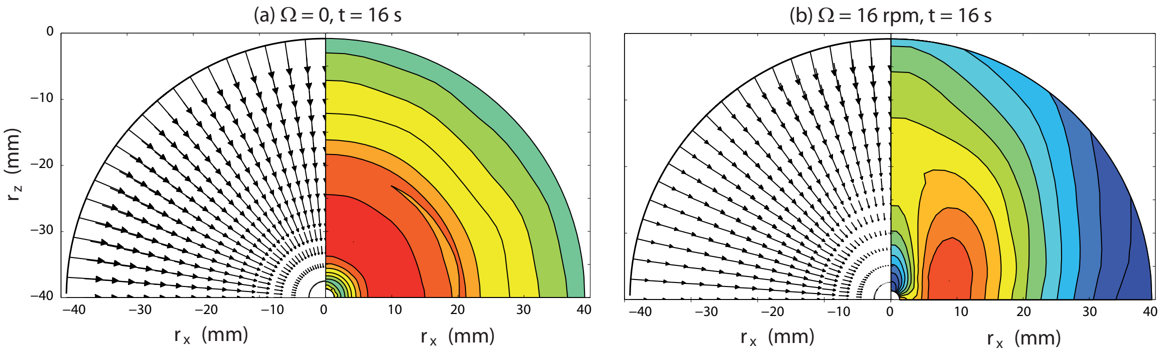

Figure 1: Maps of the energy flux density, in the separation space $\mathbf{r}$. (a) without rotation; (b) with rotation, at a time corresponding to 4.3 frame rotations. The vertical axis z corresponds to the rotation axis. Left: raw vector field $\mathbf{F}(\mathbf{r})$, showing the direct energy cascade from large to small scales. Right: Energy flux $-\nabla \cdot \mathbf{F}(\mathbf{r})$. The annular or vertically elongated region where the flux is approximately constant (red area) corresponds to the inertial range. Adapted from Ref. [8].

negative and approximately constant (red area in figure 1), becomes vertically elongated in the rotating case as time proceeds. This spatial structure is consistent with a growing anisotropy of the turbulence. Indeed, neglecting the viscous term in the KHM equation (1), the vertically elongated region where $\nabla \cdot \mathbf{F} < 0$ induces a stronger reduction of the velocity correlation R along x than along z, resulting in a relative growth of the vertical correlation along z. The striking result here is that the strongly anisotropic energy flux $\nabla \cdot \mathbf{F}$ originates from an almost purely radial, but angle-dependent, density flux $\mathbf{F}$. In particular, setting the polar component of $\mathbf{F}$ to zero yields an almost unchanged flux map $\nabla \cdot \mathbf{F}$.

These first results of the energy transfers in the physical space are a useful alternative to the more classical description in the spectral space [3], and shed new light on the anisotropy growth of decaying rotating turbulence.

References

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