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Experimental study of the instability of a film flowing down a vertical fiber

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Abstract. The wavy dynamics of a viscous film flowing axisymmetrically on a vertical fiber is characterized experimentally. The study of the initial response of the flow to natural noise shows a well-defined transition between a convective and an absolute instability [1]. In the convective case, disturbances of controlled frequencies have been applied at inlet. The flow responds to inlet excitation at frequencies lower than a well-defined cut-off frequency. A good agreement has been found with the linear stability analysis and with the solution to a two-equation model [2] in the nonlinear regime.

Fiber coating is of practical importance and occurs in many technical procedures, for example in the manufacturing process of optical fibers. The instability of a film flowing axisymmetrically on a fiber under the action of gravity has therefore attracted a considerable attention over the recent past years [3–5]. However, available experimental data [3, 4] are restricted to the fully developed non linear patterns far from inlet and to very viscous flow for which inertia can be legitimately neglected. In our experiments, parameters are chosen such as to observe the predominance of the Rayleigh-Plateau instability mechanism when inertia is weak and also the effects on the flow patterns of the Kapitza instability mode at moderate Reynolds numbers Re. We analyse the linear primary instability in the case of naturally excited flows and study the response of the flow to a periodic perturbation at inlet.

In our experimental set-up, a Rhodorsil silicon oil v50 (density $\rho = 963\,\mathrm{kg/m^3}$, kinematic viscosity $\nu = 50 \cdot 10^{-6}\,\mathrm{m^2/s}$ and surface tension $\gamma = 20.8 \cdot 10^{-3}\,\mathrm{N/m}$ at $25\,^\circ\mathrm{C}$) flows on a nylon fibre maintained vertically with a weight. The inlet flow rate is controlled by varying the gap separating two cone-shaped parts of an entrance valve. This original design ensures the axisymmetry of the base flow and limits the entrance noise (film initially uniform with thickness fluctuations of $10^{-3}\%$). This set-up allows us to vary the fibre radius ($0.23\,\mathrm{mm} < R < 1.5\,\mathrm{mm}$) and the flow rate ($0.01\,\mathrm{g.s^{-1}} < q < 3\,\mathrm{g.s^{-1}}$ corresponding to a range of uniform film thickness $0.5R < h_N < 3R$). We study the system response to a white noise (ambient noise) and to periodic perturbations with a wide range of forcing frequencies f.

The instability is mainly induced by inertial effects as observed on liquid films falling over planar substrates (Kapitza instability mechanism, see Ref. [6] for a review) except for both small fibre radii and low flow rates; the instability mechanism is then a capillary Rayleigh-Plateau mechanism. In the first case, we observe solitary waves with a rather long tail and a sharp front (figure 4(a)), whereas in the second case rather symmetrical drops are observed (figure 1(a)).

We first consider thin fibres ($0.23\,\mathrm{mm} < R < 0.475\,\mathrm{mm}$) for which the Rayleigh-Plateau instability is dominant and study the naturally excited patterns. Short after the inlet, the film of uniform thickness breaks up spontaneously into a primary wavetrain with a frequency close to the most spatially amplified frequency f_M predicted by the linear stability analysis. Further downstream, a secondary instability is observed in most cases. The primary wavetrain

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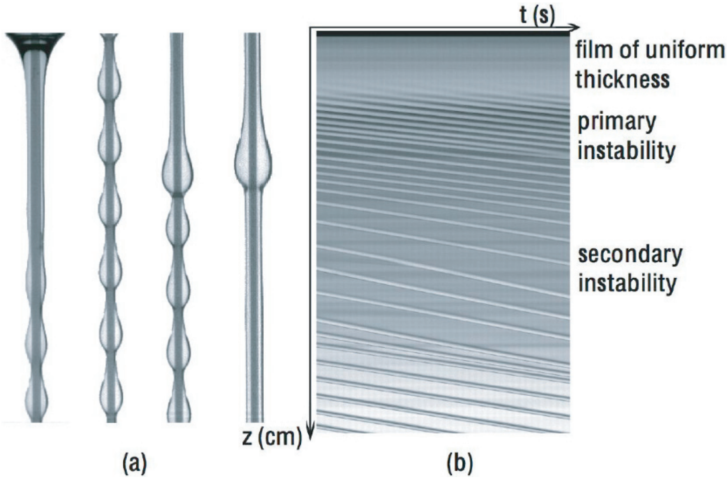

Fig. 1. Results for a fibre of radius $R = 0.25$ mm: (a) Snapshots of the liquid film at different heights, with a thickness of the uniform film of $h = 0.15$ mm (b) Spatio-temporal diagram.

is then disorganised leading to a disordered regime (see figure 1). We first consider the primary instability for which two different scenarios are observed. At low flow rate, the system exhibits a self-sustained dynamics; the spatial development of the wavetrains takes over the advection of the waves which leads to the selection of a regular pattern with a well defined intrinsic frequency. The instability is said to be absolute. At larger flow rates, a noise driven dynamics due to the advection of the growing waves by the flow is observed. The instability is then convective. We identify the critical flow rate at which the transition between the regular and the noise driven instability occurs by comparing the growth time τ_m of the instability (time of growth of the most amplified wave) and the characteristic time τ for the wave to be advected for a distance of order the wavelength of the instability λ. Kliakhandler [3] and Craster and Matar [4] have derived evolution equations for the thickness of a liquid film falling down a vertical fibre. In that case, the wavenumber leading to the maximal growth rate is given by $k_m = 1/\sqrt{2}(R+h_N)$ corresponding to the wavelength $\lambda = 2\pi\sqrt{2}(R+h_N)$. The ratio between the two characteristic times gives

$$\frac{\tau_m}{\tau} = \frac{4\kappa^2(R+h_N)^5}{u_N h_N R}\frac{2u_i}{\sqrt{2}(R+h_N)} \tag{1}$$

where $\kappa^{-1} = \sqrt{\gamma/\rho g}$ is the capillary length, u_i is the speed of the fluid at the interface and u_N is the mean velocity in the fluid based on the flow rate. Writing $\alpha_N = h_N/R$, this leads to the condition for $\tau \sim \tau_m$

$$\frac{\alpha_N}{(u_i/u_N)(1+\alpha_N)^4} = b(\kappa R)^2 \tag{2}$$

where u_i/u_N is a function of α_N. The constant b ($b \simeq 1.23$) is determined through careful calculation of the A/C transition [1,7]. We can then determine for each radius the critical thickness above which $\tau \lesssim \tau_m$, i.e. the waves are advected faster than they grow and the instability is convective. For smaller thickness, $\tau \gtrsim \tau_m$, the waves grow before they are advected, the instability being then absolute. The C/A boundary of the transition from convective to absolute instability regions in the parameter plane (α_N, κR) obtained from relation (2) is shown in figure 2. The experimental data are depicted as crosses (irregular wavetrains) and dots (regular wavetrains). The C/A boundary is in good agreement with the experimental transition from irregular to regular wavetrains and with the results of a stability analysis based on the linearized Navier-Stokes equations [1]. We conclude that the intrinsic, self-sustained dynamics leading to the formation of regular wavetrains indeed corresponds to an absolute linear instability of the uniform and steady base flow, whereas irregular wavetrains are triggered by inlet noise and correspond to the characteristic response of a convectively unstable flow.

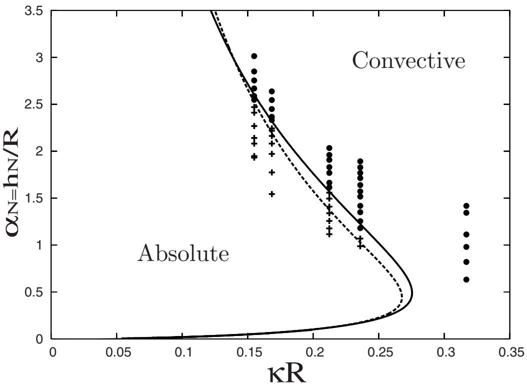

Fig. 2. Convective/Absolute transition: experimental data and predicted boundaries. The dashed line corresponds to the transition obtained with relation 2 and the thick line corresponds to the analysis of the Navier-Stokes equations.

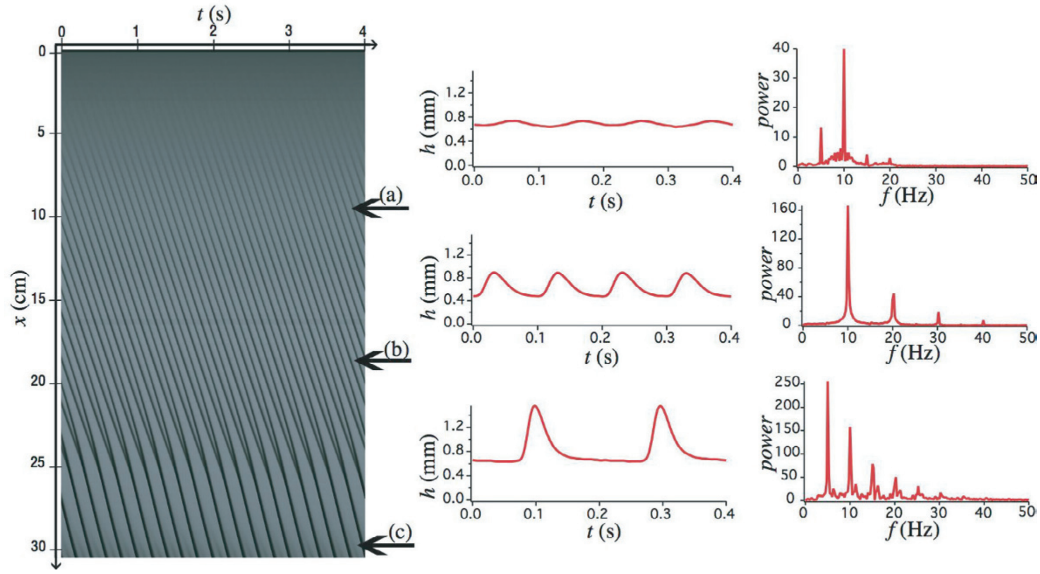

Fig. 3. Spatial response of the film to a periodic perturbation at $f = 5\,\text{Hz} \sim f_M/2$ on a fibre of radius $R = 1.5\,\text{mm}$ for a flow rate $q = 0.63\,gs^{-1}$ (corresponding to a film of uniform thickness $h_N = 0.87\,\text{mm}$). From left to right: spatiotemporal diagram; temporal evolutions of the thickness at (a) $x = 8\,\text{cm}$, (b) $x = 18\,\text{cm}$ and (c) $x = 30\,\text{cm}$; corresponding spectra.

We then consider thicker fibres ($R = 0.475\,\text{mm}$ and $R = 1.5\,\text{mm}$), whose radii are comparable to the capillarity length ($\kappa^{-1} = 1.5\,\text{mm}$). In this case, the curvature effects are dominated by the inertial effects and the response of the film to inlet perturbations is similar to the well documented case of a film flowing on a vertical plate. The system behaves as a noise amplifier [8]. We identify the cut-off frequency as the frequency for which the forcing stops affecting the dynamics of the system. For lower forcing frequencies, we obtain a periodic train of stationary saturated waves (travelling waves). An example of the response of the film to periodic perturbations is shown on figure 3. The system selects the most unstable frequency, namely the most spatially amplified harmonics. A coalescence occurs at a fixed location leading to travelling waves with periodicity corresponding to the inlet forcing. The experimental data (frequency, speed, shape and thickness of the waves) compare well to the travelling wave solutions of a model consisting in two evolution equations for the flow rate q and the film thickness h [2]. Some results obtained with large fibres are summarised on figure 4.

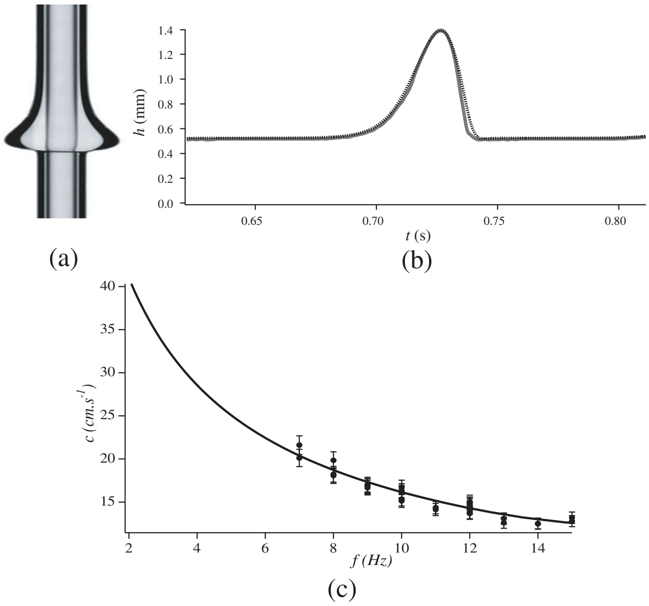

Fig. 4. Results for a fibre of radius $R = 0.475\,\mathrm{mm}$ for a flow rate $q = 0.3\,\mathrm{g.s}^{-1}$ (corresponding to a film of uniform thickness $h_N = 0.8\,\mathrm{mm}$) (a) Snapshot of a travelling wave at $f = 10\,\mathrm{Hz}$ (b) Experimental (plain line) and simulated (dashed line) wave profile for $f = 8\,\mathrm{Hz}$. (c) Variation of the waves speed with the forcing frequency (dots correspond to the experimental data, line corresponds to the solution of the model).

To conclude, the spatial response of the film to inlet controlled periodic perturbations have been characterised when the Rayleigh-Plateau and the Kapitza instability mechanisms are relevant. In this preliminary report, the emphasis has been put on the linear and nonlinear development of the primary wavetrain. We now aim at a more quantitative characterisation of the secondary instabilities that disorganise the flow.

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