Penetration of a projectile by impact into a granular medium

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Summary. When a solid sphere drops in a granular medium, an impact crater is created. The penetration of the projectile in the granular bed as a function of the impact energy is evaluated theoretically using a simple model including a friction law between the projectile and the grains, a viscous dissipation in the bed and a force from the collisions between the projectile and the granular material. This model is observed to be in agreement with our quasi-2D experimental results and suggests that the penetration depth is a power law of the total drop distance.

For millions of years, meteorites have crashed on Earth, ejecting rocks at huge velocity and creating enormous craters. Although these collisions are happened during a short time, they may have biological and geophysical aftermaths. That is why the knowledge of the projectile and the impacted medium is significant. The creation of impact craters is an intricate dynamics and the penetration of a projectile needs to be modelled with conservative law to reproduce the behavior of a granular medium. When a solid projectile impacts a granular layer, a crater is created and the projectile stops at a penetration depth δ. The morphology and the scaling of this crater depend on the granular medium and many features of the impacting projectile (diameter, density, impact velocity...) [1].

Recent experiments have brought out the dependance of the penetration depth δ with experimental parameters such as the projectile diameter d and density ρ, and the drop height [2, 3]. For a spherical projectile impacting a 3D granular layer, the evolution of the penetration δ with these parameters is usually described by power laws. Introducing the free fall height h and the total drop distance $H = h + \delta$, the depth is generally adjusted by a power law of the form $\delta \propto H^{1/3}$. In a recent work, Katsuragi and Durian [4] propose that the total force on the projectile results from the sum of three contributions: the gravity, a Coulomb solid friction and an inertial drag, such as

$$m\frac{dv}{dt} = mg - F_f - \alpha_c v^2. \qquad (1)$$

From their experimental measurements, they suggest that the friction force F_f can be reduced to a simple linear behavior, $F_f = \alpha_f |z|$, where α_f mainly depends on the projectile diameter d and density ρ, and the friction coefficient in the granular medium.

Otherwise, experiments and numerical simulations have been undertaken in a two-dimensional case, to study more specifically the dynamics of the projectile during a crater formation [5, 6, 7]. In particular, they show that the time taken for a projectile to slow to a stop in the granular medium is independent of its velocity at impact. Moreover, the mean drag force on the projectile dropped into the granular medium is observed to be constant during most of the projectile's trajectory and proportional to the impact velocity v suggesting that fluid-like properties might be important.

In this paper, we chose to focus on the penetration dynamics of a projectile in a granular medium in a quasi-2D experiment. Here we are going to expose an other model which describes our results.

1 Experimental setup and procedure

A packing of 1 mm glass beads (density $\rho_g = 2.5 \times 10^3\,\mathrm{kg\ m^{-3}}$) is confined into a vertical Hele-Shaw cell consisting of two parallel glass plates separated by a gap of 15 mm. In order to have reproducible measurements, the initial grain piling of height 56 mm is gently stirred with a thin rod before each experiment, allowing us to consider that a random close packing has been reached (we checked that measurements are well reproducible in our experiments).

A cylindrical steel projectile (density $\rho = 7.7 \times 10^3\,\mathrm{kg\ m^{-3}}$, mass $m \simeq 8.9\,\mathrm{g}$) of diameter $d = 10\,\mathrm{mm}$ is hold initially by a magnet at a distance h above the granular surface. This apparatus allows us to drop the projectile without any initial velocity and spinning motion. The length of the projectile is a little smaller than the gap width to reduce the friction with the plates, and to prevent beads to be wedged between the glass plates and the projectile during the collision process. The impact speed is tuned by varying the drop height h from 75 to 305 mm corresponding to impact velocities ranging from 1.2 to $2.5\,\mathrm{m\ s^{-1}}$.

The dynamical properties of the impact are analyzed by means of a high speed video camera at a frame rate of 2000 Hz. Using image analysis, the position of the projectile is then extracted during the free fall stage and as it penetrates inside the granular medium. In the following, the presented results correspond to an average over ten experiments to reduce the fluctuations of the measurements.

2 Experimental results

Figure 1(a) displays the position of the projectile as a function of time for three different drop heights h (note that $t = 0$ corresponds to the beginning of the penetration phase). Despite the fact that the projectile undergoes two very different phases – the free fall motion in the air and the penetration inside the granular layer – the transition does not appear clearly regarding the position z. As observed in Fig. 1(a), z tends towards a finite value δ (the penetration depth), all the more higher than the drop height is important. The curve is then derived numerically to get the variation of the velocity v as a function of time [Fig. 1(b)]. After a first stage where the velocity increases linearly during the free fall phase with a slope equals to gravity acceleration g, the velocity decreases abruptly with a discontinuity in

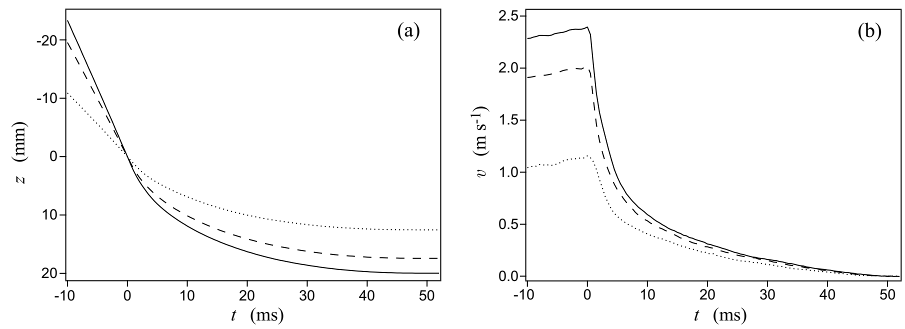

Fig. 1. (a) Position z and (b) velocity v of the projectile versus time for three different drop heights: (· ·) h=75 mm, (– –) h=215 mm, (—) h=305 mm.

acceleration as soon as the projectile hits the granular surface. These behaviors are in agreement with those observed both in two and three dimensional experiments reported elsewhere [4, 5]. Does the model proposed by Katsuragi and Durian [see Eq. (1)] to describe the forces at work during the impact reproduce the behaviors observed in our 2D experiment ?

3 Modelling

We did not succeed to fit our results with the model of Katsuragi and Durian [4] as if something missed. So we add another friction force of viscous type so that the equation of the projectile during the penetration phase is

$$m\frac{d^2z}{dt^2} = mg - \alpha_f|z| - \alpha_v v - \alpha_c v^2. \quad (2)$$

The term $\alpha_f|z|$ represents a solid friction, without which the projectile would not stop at a finite penetration depth. The term $\alpha_c v^2$ can be representative of a dynamic frictional force which includes collision of the grains, as already presented in Katsuragi and Durian's work [4]. The term $\alpha_v v$ has been used in the pioneering work of Allen *et al.* for the penetration of a bullet in sand [8] and is now justified by the recent dense granular rheology where the friction coefficient increases linearly with the velocity [9].

The solution of this model obtained with a numerical solver is displayed in Fig. 2 and compared with our experimental measurements for three different drop heights. The numerical coefficients corresponding to the best fit of the experimental curves for the position and the velocity are α_f=6 kg s^{-2}, α_c=0.7 kg m^{-1} and α_v=0.58 kg s^{-1}. We can directly compare these values with the ones of Katsuragi and Durian [4]; in their 3D experiment, they estimate $\alpha_f = 7.2\ 10^{-3}$ kg s^{-2} and α_c=0.79 kg m^{-1} using a spherical projectile of mass 62.9 g and a granular layer composed of spherical glass beads (diameter ranges from 250 to 350 μm). We obtain the same order of magnitude for α_c, whereas the value of α_f is completely different. The

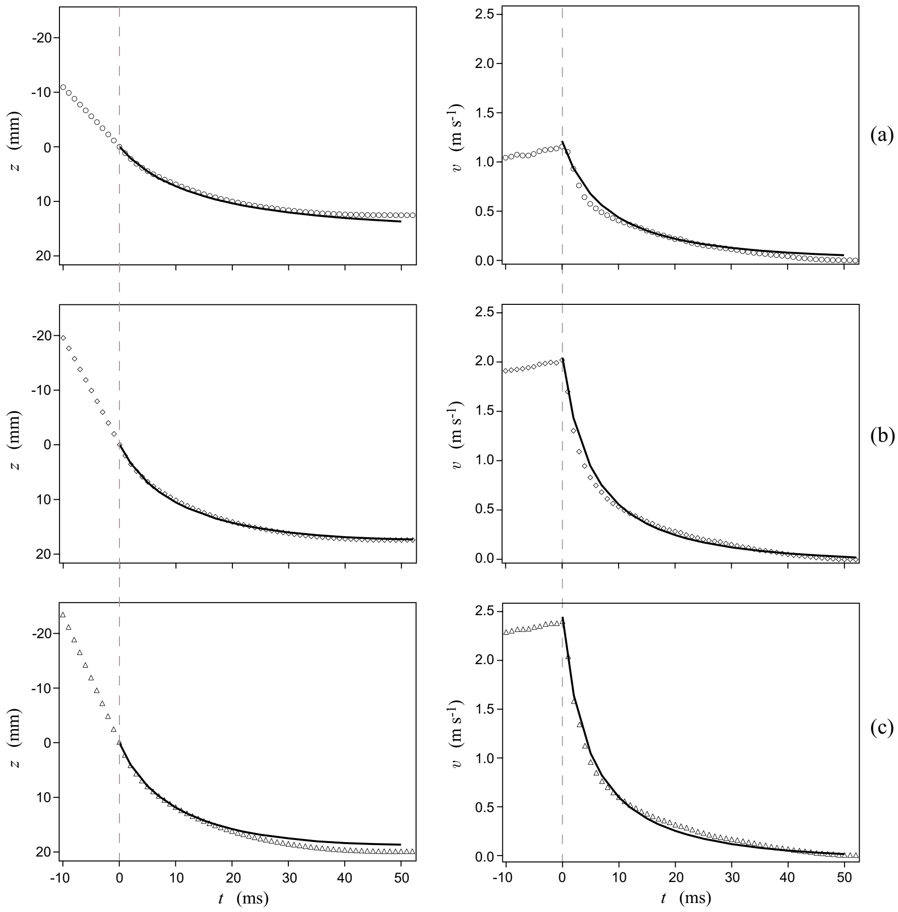

Fig. 2. Position z and velocity v of the projectile as a function of time for (a) h=75 mm, (b) h=215 mm, (c) h=305 mm. (—) Best fit of experimental data with numerical values α_f=6 kg s^{-2}, α_c=0.7 kg s^{-1} and α_v=0.58 kg m^{-1}. (- -) Time at which the impact occurs.

differences might be explained by the fact the experimental conditions – such as the bead and projectile diameters or the 2D/3D nature of the experiment – are different.

Note that each parameter has its own action field. At the beginning of the penetration, where the projectile velocity is important, the term $\alpha_c v^2$ dominates the other terms. At the end of the penetration, the projectile slows down and both terms $\alpha_c v^2$ and $\alpha_v v$ become negligible compared with $\alpha_f |z|$. Our model fits the experimental results reasonably even if small differences are still existing. Note also that α_f, α_v and α_c depend on many physical parameters (such as density, geometry...) but the study of such dependencies is far from the scope of this paper.

Moreover, our model allows one to extract the penetration depth δ for a given drop height h. The plot of the penetration depth versus the total drop distance H

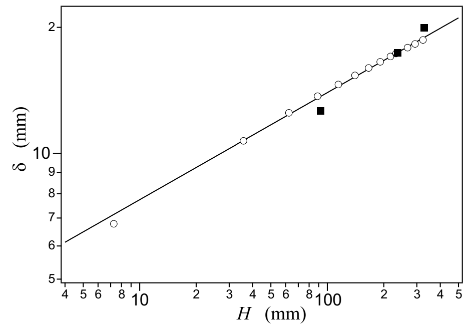

Fig. 3. Penetration depth δ versus the total drop distance H. (■) Experimental measurements; (○) numerical values; (—) the adjustment is a power law of the form $\delta = AH^{\lambda}$ with $\lambda \simeq 1/4$ and $A \simeq 4.3$ S.I..

(where $H = h + \delta$), show that the numerical results can be adjusted by a power law $\delta = AH^{\lambda}$ with $\lambda \simeq 1/4$ and $A \simeq 4.3$ S.I. (Fig. 3). These results display the same behavior as the 3D experimental results of Ambroso, De Bruyn and coworkers [2, 3], but with a smaller exponent, as they found $\lambda \simeq 1/3$ in 3D. This may be attributed to confinement effects in our Hele-Shaw cell.

4 Conclusion and Outlook

In summary, we have presented the penetration of a projectile impacting in a granular layer in a quasi-2D experiment. First, we succeed to find that the penetration depth δ follows a scaling law similar to the 3D experimental results of the form $\delta = AH^{\lambda}$ but with a smaller exponent λ probably due to confinement effects. Then, the dynamics of penetration is well adjusted by a model where many forces act: the gravity, a solid friction force $\alpha_f|z|$, a viscous force $\alpha_v v$ and a dynamic frictional force $\alpha_c v^2$.

Ultimately, additional experiments will be realized by modifying the granular layer and modifying the projectile (with different diameters d and densities ρ) in order to test the validity of the model and to observe the dependance of the three coefficients α_f, α_v and α_c with these parameters.

References

1. Walsh A. M., Holloway K. E., Habdas P. and De Bruyn J. R. (2003) Phys. Rev. Lett. 91:104301
2. Ambroso M. A., Santore C. R., Abate A. R. and Durian D. J. (2005) Phys. Rev. E 71:051305
3. De Bruyn J. R. and A. M. Walsh (2004) Can. J. Phys. 82:439–446
4. Katsuragi H. and Durian D. J. (2007) Nature Phys. 3:420–423
5. Ciamarra M. P., Lara A. H., Lee A. T., Goldman D. I., Vishik I. and Swinney H. L. (2004) Phys. Rev. Lett. 92:194301

6. Tsimring L. S. and Volfson D. (2005) Modeling of impact cratering in granular media. In Powders & Grains 2005, R. García-Rojo, H.J. Herrmann, and S. McNamara (Eds.), A.A. Balkema, Rotterdam, 1215–1223
7. Hou M., Peng Z., Liu R. and Lu K. (2005) Phys. Rev. E 72:062301
8. Allen W. A., Mayfield E. B. and Morrison H. L. (1956) J. Appl. Phys. 28(3):370–376
9. GDR MiDi (2004) Eur. Phys. J. E 14:341–365